This is the fifth version of the catalog of NP optimization problems.
Please send any comment or suggestion to one of the two authors.

A compendium of NP optimization problems

P. Crescenzi[1] and V. Kann[2]

[1] Dipartimento di Scienze dell'Informazione, Università degli Studi di Roma "La Sapienza", Via Salaria 113, 00198 Rome, Italy. E-mail: piluc@dsi.uniroma1.it
[2] Department of Numerical Analysis and Computing Science, Royal Institute of Technology, SE-100 44 Stockholm, Sweden. E-mail: viggo@nada.kth.se

April 21, 1997

The compendium is also available in WWW as `http://www.nada.kth.se/theory/problemlist.html`

Summary. Due to the fact that no NP-complete problem can be solved in polynomial time (unless P=NP), many approximability results (both positive and negative) of NP-hard optimization problems have appeared in the technical literature. In this compendium, we collect a large number of these results.

1. Introduction

In the following we refer to standard complexity classes (see [Johnson, 1990]). We recall that a function $t(n)$ is 'quasi-polynomial' if a constant c exists such that $t(n) \leq n^{\log^c n}$ and we denote by QP, QNP, and QR the analogues of the usual complexity classes in the quasi-polynomial time domain.

1.1. NPO Problems: Definitions and Preliminaries

The basic ingredients of an optimization problem are the set of instances or input objects, the set of feasible solutions or output objects associated with any instance, and the measure defined for any feasible solution. On the analogy of the theory of NP-completeness, we are interested in studying a class of optimization problems whose feasible solutions are short and easy-to-recognize. To this aim, suitable constraints have to be introduced. We thus give the following definition.

Definition 1. *An* NP optimization problem A *is a fourtuple* $(I, sol, m, goal)$ *such that*

1. *I is the set of the* instances *of A and it is recognizable in polynomial time.*
2. *Given an instance x of I, $sol(x)$ denotes the set of* feasible solutions *of x. These solutions are short, that is, a polynomial p exists such that, for any $y \in sol(x)$, $|y| \leq p(|x|)$. Moreover, it is decidable in polynomial time whether, for any x and for any y such that $|y| \leq p(|x|)$, $y \in sol(x)$.*

3. *Given an instance x and a feasible solution y of x, $m(x, y)$ denotes the positive integer* measure *of y. The function m is computable in polynomial time and is also called the* objective *function.*
4. *goal* $\in \{\max, \min\}$.

The *class* NPO is the set of all NP optimization problems.

The goal of an NPO problem with respect to an instance x is to find an *optimum solution*, that is, a feasible solution y such that

$$m(x, y) = goal\{m(x, y') \ : \ y' \in sol(x)\}.$$

In the following, *opt* will denote the function mapping an instance x to the measure of an optimum solution.

An NPO problem is said to be *polynomially bounded* if a polynomial q exists such that, for any instance x and for any solution y of x, $m(x, y) \leq q(|x|)$. The *class* NPOPB is the set of polynomially bounded NPO problems.

1.2. Approximate Algorithms and Approximation Classes

It is well-known that if an NPO problem can be solved in polynomial time, then its corresponding decision problem can also be solved in polynomial time. As a consequence, if P $\neq$ NP, then any NPO problem whose corresponding decision problem is NP-complete is not solvable in polynomial time. In these cases we sacrifice optimality and start looking for approximate solutions computable in polynomial time.

Definition 2. *Let A be an* NPO *problem. Given an instance x and a feasible solution y of x, we define the* performance ratio of y with respect to x *as*

$$R(x, y) = \max\left\{\frac{m(x, y)}{opt(x)}, \frac{opt(x)}{m(x, y)}\right\}.$$

The performance ratio is always a number greater than or equal to 1 and is as close to 1 as y is close to the optimum solution.

Definition 3. *Let A be an* NPO *problem and let T be an algorithm that, for any instance x of A, returns a feasible solution $T(x)$ of x. Given an arbitrary function $r : N \rightarrow (1, \infty)$, we say that T is an $r(n)$*-approximate algorithm for A *if, for any instance x, the performance ratio of the feasible solution $T(x)$ with respect to x verifies the following inequality:*

$$R(x, T(x)) \leq r(|x|).$$

If an NPO problem admits an $r(n)$-approximate polynomial-time algorithm we say that it is approximable within $r(n)$.

Definition 4. *An* NPO *problem A belongs to the* class APX *if it is approximable within ε, for some constant $\varepsilon > 1$.*

Definition 5. *Let A be an* NPO *problem. An algorithm T is said to be an* approximation scheme *for A if, for any instance x of A and for any rational $\varepsilon > 1$, $T(x, \varepsilon)$ returns a feasible solution of x whose performance ratio is at most ε.*

Definition 6. *An* NPO *problem A belongs to the* class PTAS *if it admits a polynomial-time approximation scheme, that is, an approximation scheme whose time complexity is bounded by $q(|x|)$ where q is a polynomial.*

Observe that the time complexity of an approximation scheme in the above definition may be of the type $2^{1/(\varepsilon-1)}p(|x|)$ or $|x|^{1/(\varepsilon-1)}$ where p is a polynomial. Thus, computations with ε values very close to 1 may turn out to be practically unfeasible. This leads us to the following definition.

Definition 7. *An* NPO *problem A belongs to the* class FPTAS *if it admits a* fully *polynomial-time approximation scheme, that is, an approximation scheme whose time complexity is bounded by $q(|x|, 1/(\varepsilon-1))$ where q is a polynomial.*

Clearly, the following inclusions hold:

$$\text{FPTAS} \subseteq \text{PTAS} \subseteq \text{APX} \subseteq \text{NPO}.$$

It is also easy to see that these inclusions are strict if and only if $\mathrm{P} \neq \mathrm{NP}$.

1.3. Completeness in Approximation Classes

In this section we define a natural approximation preserving reducibility and introduce the notion of completeness both in NPO and in APX.

Definition 8. *Let A and B be two* NPO *problems. A is said to be* PTAS-reducible *to B, in symbols $A \leq_{\mathrm{PTAS}} B$, if three functions f, g, and c exist such that:*

1. *For any $x \in I_A$ and for any rational $\varepsilon \in (1, \infty)$, $f(x, \varepsilon) \in I_B$ is computable in time polynomial with respect to $|x|$.*
2. *For any $x \in I_A$, for any $y \in sol_B(f(x, \varepsilon))$, and for any rational $\varepsilon \in (1, \infty)$, $g(x, y, \varepsilon) \in sol_A(x)$ is computable in time polynomial with respect to both $|x|$ and $|y|$.*
3. *$c : (1, \infty) \to (1, \infty)$ is computable and invertible.*
4. *For any $x \in I_A$, for any $y \in sol_B(f(x, \varepsilon))$, and for any rational $\varepsilon \in (1, \infty)$,*

$$R_B(f(x, \varepsilon), y) \leq c(\varepsilon) \textit{ implies } R_A(x, g(x, y, \varepsilon)) \leq \varepsilon.$$

Remark 1. In [Papadimitriou and Yannakakis, 1991] a different kind of reducibility between optimization problems is defined which is called L-reducibility. It is possible to prove that, within APX, the L-reducibility is a restriction of the PTAS-reducibility [Crescenzi, Silvestri, and Trevisan, 1994]. More recently, another restriction of the PTAS-reducibility, called E-reducibility, has been introduced in [Khanna, Motwani, Sudan, and Vazirani, 1994].

Proposition 1. *If $A \leq_{\mathrm{PTAS}} B$ and $B \in$ APX (respectively, $B \in$ PTAS), then $A \in$ APX (respectively, $A \in$ PTAS).*

Definition 9. *A problem $A \in$ NPO is* NPO-complete *if, for any $B \in$ NPO, $B \leq_{\mathrm{PTAS}} A$.*

Definition 10. *A problem $A \in$ NPO is* APX-hard *if, for any $B \in$ APX, $B \leq_{\mathrm{PTAS}} A$. An* APX*-hard problem is* APX-complete *if it belongs to* APX.

From Proposition 1 it immediately follows that if an NPO problem A is NPO-complete (respectively, APX-hard) then it does not belong to APX (respectively, PTAS). It is also possible to prove that if A is NPO-complete (respectively, NPOPB-complete) then it cannot be approximated within 2^{n^ε} (respectively, n^ε) for some $\varepsilon > 0$.

The syntactically defined classes MAX SNP (containing e.g. MAXIMUM 3-SATISFIABILITY and MAXIMUM CUT) and MAX NP (containing e.g. MAXIMUM SATISFIABILITY) were defined in [Papadimitriou and Yannakakis, 1991]. Recently was shown that the closures of these classes under PTAS-reduction were identical to APX [Khanna, Motwani, Sudan, and Vazirani, 1994] and [Crescenzi and Trevisan, 1994]. In the compendium we therefore always use the term APX-*complete* instead of MAX SNP-*complete* and APX-*hard* instead of MAX SNP-*hard*.

The classes MAX PB and MIN PB consisting of the polynomially bounded maximization and minimization problems, respectively, were defined in [Kolaitis and Thakur, 1994]. The closures of these classes under PTAS-reduction have recently been shown to be identical to NPOPB [Crescenzi, Kann, Silvestri, and Trevisan]. In the compendium we therefore always use NPOPB-*complete* instead of MAX PB-*complete* and MIN PB-*complete*.

1.4. A list of NPO problems

The list contains more than 200 entries. A typical entry consists of eight parts: the first four parts are mandatory while the last four parts are optional.

1. The problem name that also specifies the goal of the problem.
2. The definition of the instances of the problem.
3. The definition of the feasible solutions of the problem.
4. The definition of the measure of a feasible solution.
5. A 'good news' part that contains the best approximation result for the problem.
6. A 'bad news' part that contains the worst approximation negative result for the problem.
7. A section of additional comments.
8. A reference to the 'closest' problem appearing in the list published in [Garey and Johnson, 1979].

The list is organized according to subject matter as done in [Garey and Johnson, 1979]. In particular the entries are divided into the following twelve categories:

GT Graph theory: 51 entries.
ND Network design: 59 entries.
SP Sets and partitions: 11 entries.
SR Storage and retrieval: 10 entries.
SS Sequencing and scheduling: 20 entries.
MP Mathematical programming: 17 entries.
AN Algebra and number theory: 1 entry.
GP Games and puzzles: 2 entry.
LO Logic: 13 entries.
AL Automata and language theory: 5 entries.
PO Program optimization: 1 entry.
MS Miscellaneous: 14 entries.

We have ignored problems with too obscure definitions and problems for which the membership in NP was not guaranteed.

1.5. Improving the compendium

We ask everybody to help us in correcting, improving, and enlarging the problem list. The easiest way of doing this is by using WWW. Forms for reporting new problems, new results and errors are available.

The address is `http://www.nada.kth.se/theory/problemlist.html`

Graph Theory

Covering and Partitioning

GT1. MINIMUM VERTEX COVER

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A vertex cover for G, i.e., a subset $V' \subseteq V$ such that, for each edge $(u, v) \in E$, at least one of u and v belongs to V'.
MEASURE: Cardinality of the vertex cover, i.e., $|V'|$.

Good News: Approximable within $2 - \frac{\log\log|V|}{2\log|V|}$ [Monien and Speckenmeyer, 1985] and [Bar-Yehuda and Even, 1985].
Bad News: APX-complete [Papadimitriou and Yannakakis, 1991].
Comment: Transformation from bounded MAXIMUM 3-SATISFIABILITY. Not approximable within 1.1666 [Håstad, 1997]. Admits a PTAS for planar graphs [Baker, 1994] and for unit disk graphs [Hunt III et al., 1994b]. Variation in which each vertex has a nonnegative weight and the objective is to minimize the total weight of the vertex cover is approximable within $2 - \log\log|V|/2\log|V|$ on general graphs [Bar-Yehuda and Even, 1985] and admits a PTAS for planar graphs [Baker, 1994]. Variation in which the degree of G is bounded by a constant B for $B \geq 3$ is APX-complete [Papadimitriou and Yannakakis, 1991] and [Alimonti and Kann, 1997] and not approximable within 1.1666 for a sufficiently large B [Clementi and Trevisan, 1996]. For $B = 3$ it is approximable within $7/6+\varepsilon$ [Berman and Fujito, 1995], for general B within $2 - \log B/B(1 + \varepsilon)$ and within $3/2$ for 6-claw-free graphs (where no independent set of size 6 exists in any neighbour set to any vertex) [Halldórsson, 1995a]. Variation in which the problem is restricted to 'dense' graphs is APX-complete [Clementi and Trevisan, 1996]. The generalization to k-hypergraphs, for $k \geq 2$, is approximable within k [Hochbaum, 1982]. If the vertex cover is required to induce a connected graph, the problem is approximable within 2 [Arkin, Halldórsson, and Hassin, 1993]. If the graph is edge-weighted, the solution is a closed walk whose vertices form a vertex cover, and the objective is to minimize the sum of the edges in the cycle, the problem is approximable within 5.5 [Arkin, Halldórsson, and Hassin, 1993].
The constrained variation in which the input is extended with a positive integer k and a subset S of V, and the problem is to find the vertex cover of size k that contains the largest number of vertices from S, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
The maximization variation in which the input is extended just with a positive integer k, and the problem is to find k vertices that cover as many edges as possible, does not admit a PTAS [Petrank, 1994].
Garey and Johnson: GT1

GT2. MINIMUM DOMINATING SET

INSTANCE: Graph $G = \langle V, E \rangle$.
SOLUTION: A dominating set for G, i.e., a subset $V' \subseteq V$ such that for all $u \in V - V'$ there is a $v \in V'$ for which $(u, v) \in E$.
MEASURE: Cardinality of the dominating set, i.e., $|V'|$.

Good News: Approximable within $1+\log|V|$ since the problem is a special instance of MINIMUM SET COVER [Johnson, 1974].

Bad News: Not in APX [Lund and Yannakakis, 1994] and [Bellare, Goldwasser, Lund, and Russell, 1993].

Comment: Equivalent to MINIMUM SET COVER under L-reduction [Kann, 1992b]. See MINIMUM SET COVER for more comments. Not approximable within $(1-\varepsilon)\ln|V|$ for any $\varepsilon > 0$, unless NP$\subset$ DTIME$(n^{\log\log n})$ [Feige, 1996]. If it is NP-hard to approximate within $\Omega(\log n)$, then it is complete for the class of log-approximable problems [Khanna, Motwani, Sudan, and Vazirani, 1994]. Admits a PTAS for planar graphs [Baker, 1994] and for unit disk graphs [Hunt III et al., 1994b]. Variation in which the degree of G is bounded by a constant B is APX-complete [Papadimitriou and Yannakakis, 1991] and is approximable within $\sum_{i=1}^{B+1}\frac{1}{i} - 0.433$ by reduction to MINIMUM SET COVER. If the dominating set is restricted to be connected the problem is approximable within $\ln\Delta + 3$ where Δ is the maximum degree, and within $3\ln|V|$ for the vertex weighted version [Guha and Khuller, 1996].

Garey and Johnson: GT2

GT3. MINIMUM EDGE DOMINATING SET

INSTANCE: Graph $G = \langle V, E\rangle$.

SOLUTION: An edge dominating set for G, i.e., a subset $E' \subseteq E$ such that for all $e_1 \in E - E'$ there is an $e_2 \in E'$ such that e_1 and e_2 are adjacent.

MEASURE: Cardinality of the edge dominating set, i.e., $|E'|$.

Good News: Approximable within 2 (any maximal matching).

Comment: Admits a PTAS for planar graphs [Baker, 1994] and for λ-precision unit disk graphs [Hunt III et al., 1994b].

Garey and Johnson: GT2

GT4. MINIMUM INDEPENDENT DOMINATING SET

INSTANCE: Graph $G = \langle V, E\rangle$.

SOLUTION: An independent dominating set for G, i.e., a subset $V' \subseteq V$ such that for all $u \in V - V'$ there is a $v \in V'$ for which $(u, v) \in E$, and such that no two vertices in V' are joined by an edge in E.

MEASURE: Cardinality of the independent dominating set, i.e., $|V'|$.

Bad News: NPOPB-complete [Kann, 1994b]. Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Halldórsson, 1993b].

Comment: Also called *Minimum Maximal Independence Number.* Transformation from SHORTEST PATH WITH FORBIDDEN PAIRS. Variation in which the degree of G is bounded by a constant B is APX-complete [Kann, 1992b]. Approximable within 5 for unit disk graphs [Marathe et al., 1994].

Garey and Johnson: GT2

GT5. MINIMUM GRAPH COLORING

INSTANCE: Graph $G = \langle V, E\rangle$.

SOLUTION: A coloring of G, i.e., a partition of V into disjoint sets $V_1, V_2, \ldots, V_k$ such that each V_i is an independent set for G.
MEASURE: Cardinality of the coloring, i.e., the number of disjoint independent sets V_i.

Good News: Approximable within $O\left(|V|\frac{(\log\log|V|)^2}{(\log|V|)^3}\right)$ [Halldórsson, 1993a].
Bad News: Not approximable within $|V|^{1/7-\varepsilon}$ for any $\varepsilon > 0$ [Bellare, Goldreich, and Sudan, 1995].
Comment: Also called *Minimum Chromatic Number.* Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$, unless co-RP=NP [Feige and Kilian, 1996]. If the graph is 3-colorable the problem is approximable within $O(|V|^{1/4}\log|V|)$ [Karger, Motwani, and Sudan, 1994] and [Mahajan and Ramesh, 1995], but it is not approximable within $5/3-\varepsilon$ [Khanna, Linial and Safra, 1993]. If the graph is k-colorable the problem is approximable within $O(|V|^{1-3/(k+1)}\log|V|)$ [Karger, Motwani, and Sudan, 1994]. Approximable with an absolute error guarantee of 1 on planar graphs [see e.g. Horowitz and Sahni, 1978]. Approximable within 3 but not within 1.33 for unit disk graphs [Marathe et al., 1994]. *Minimum Coloring with Defect d*, the variation where the color classes V_i may induce graphs of degree at most d, is not approximable within $|V|^{\varepsilon}$ for some $\varepsilon > 0$ [Cowen, Goddard, and Jesurum, 1997].
MINIMUM FRACTIONAL CHROMATIC NUMBER, the linear programming relaxation in which the independent sets $V_1, V_2, \ldots, V_k$ do not need to be disjoint, and in the solution every independent set V_i is assigned a nonnegative value λ_i such that for each vertex $v \in V$ the sum of the values assigned to the independent sets containing v is at most 1, and the measure is the sum $\sum \lambda_i$, is not approximable within $|V|^c$ for some constant c [Lund and Yannakakis, 1994].
The complementary maximization problem, where the number of "not needed colors", i.e. $|V|-k$, is to be maximized, is approximable within $4/3$ [Halldórsson, 1996]. The constrained variation in which the input is extended with a positive integer k, a vertex $v_0 \in V$ and a subset S of V, and the problem is to find the k-coloring that colors the largest number of vertices from S in the same way as v_0, is not approximable within $|V|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: GT4

GT6. MAXIMUM ACHROMATIC NUMBER

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: A complete coloring of G, i.e., a partition of V into disjoint sets $V_1, V_2, \ldots, V_k$ such that each V_i is an independent set for G and such that, for each pair of distinct sets V_i, V_j, $V_i \cup V_j$ is *not* an independent set.
MEASURE: Cardinality of the complete coloring, i.e., the number of disjoint independent sets V_i.

Good News: Approximable within $O(|V|/\sqrt{\log|V|})$ [Chaudhary and Vishwanathan, 1997].
Comment: Approximable within $O(\sqrt{|V|})$ for graphs of girth (length of shortest cycle) at least 6 [Chaudhary and Vishwanathan, 1997].
Garey and Johnson: GT5

GT7. Minimum Edge Coloring

Instance: Graph $G = \langle V, E \rangle$.
Solution: A coloring of E, i.e., a partition of E into disjoint sets $E_1, E_2, \ldots, E_k$ such that, for $1 \le i \le k$, no two edges in E_i share a common endpoint in G.
Measure: Cardinality of the coloring, i.e., the number of disjoint sets E_i.

Good News: Approximable within $4/3$, and even approximable with an absolute error guarantee of 1 [Vizing, 1964].
Bad News: Not approximable within $4/3-\varepsilon$ for any $\varepsilon > 0$ [Holyer, 1981].
Comment: Also called *Minimum Chromatic Index*. Apx-intermediate unless the polynomial-hierarchy collapses [Crescenzi, Kann, Silvestri, and Trevisan]. On multigraphs the problem is approximable within $1.1+(0.8/opt)$ [Nishizeki and Kashiwagi, 1990]. The maximization variation in which the input is extended with a positive integer k, and the problem is to find the maximum number of consistent vertices over all edge-colorings with k colors, is approximable within $e/(e-1)$ [Bertsimas, Teo, and Vohra, 1996], but does not admit a Ptas [Petrank, 1994].
Garey and Johnson: OPEN5

GT8. Minimum Feedback Vertex Set

Instance: Directed graph $G = \langle V, A \rangle$.
Solution: A feedback vertex set, i.e., a subset $V' \subseteq V$ such that V' contains at least one vertex from every directed cycle in G.
Measure: Cardinality of the feedback vertex set, i.e., $|V'|$.

Good News: Approximable within $O(\log |V| \log\log |V|)$ [Seymour, 1995] and [Even, Naor, Schieber, and Sudan, 1995].
Bad News: Apx-hard [Kann, 1992b].
Comment: Transformations from Minimum Vertex Cover and Minimum Feedback Arc Set [Ausiello, D'Atri, and Protasi, 1980]. On undirected graphs the problem is Apx-complete and approximable within 2, even if the vertices are weighted [Bafna, Berman, and Fujito, 1994]. The generalized variation in which the input is extended with a subset S of vertices and arcs, and the problem is to find a vertex set that contains at least one vertex from every directed cycle that intersects S, is approximable within $O(\min(\log |V| \log\log |V|, \log^2 |S|))$ on directed graphs [Even, Naor, Schieber, and Sudan, 1995] and within 8 on undirected graphs [Even, Naor, and Zosin, 1996]. All these problems are approximable within $9/4$ for planar graphs [Goemans and Williamson, 1996]. The constrained variation in which the input is extended with a positive integer k and a subset S of V, and the problem is to find the feedback vertex set of size k that contains the largest number of vertices from S, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: GT7

GT9. Minimum Feedback Arc Set

Instance: Directed graph $G = \langle V, A \rangle$.

SOLUTION: A feedback arc set, i.e., a subset $A' \subseteq A$ such that A' contains at least one arc from every directed cycle in G.
MEASURE: Cardinality of the feedback arc set, i.e., $|A'|$.

Good News: Approximable within $O(\log|V|\log\log|V|)$ [Even, Naor, Schieber, and Sudan, 1995].
Bad News: APX-hard [Kann, 1992b].
Comment: Transformation from MINIMUM FEEDBACK VERTEX SET [Even, Naor, Schieber, and Sudan, 1995]. The generalized variation in which the input is extended with a subset S of vertices and arcs, and the problem is to find an arc set that contains at least one arc from every directed cycle that intersects S, is approximable within $O(\min(\log|V|\log\log|V|, \log^2|S|))$ [Even, Naor, Schieber, and Sudan, 1995]. All these problems are approximable within 9/4 for planar graphs [Goemans and Williamson, 1996]. The constrained variation in which the input is extended with a positive integer k and a subset S of A, and the problem is to find the feedback arc set of size k that contains the largest number of arcs from S, is not approximable within $|E|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993]. The complementary problem of finding the maximum set of arcs A' such that $G' = \langle V, A'\rangle$ is acyclic is approximable within $2/(1+\Omega(1/\sqrt{\Delta}))$ where Δ is the maximum degree [Berger and Shor, 1990] and [Hassin and Rubinstein, 1994], it is APX-complete [Papadimitriou and Yannakakis, 1991], and if $|E| = \Theta(|V|^2)$ it admits a PTAS [Arora, Frieze, and Kaplan, 1996].

Garey and Johnson: GT8

GT10. MAXIMUM TRIANGLE PACKING

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: A triangle packing for G, i.e., a collection $V_1, V_2, \ldots, V_k$ of disjoint subsets of V, each containing exactly 3 vertices, such that for each $V_i = \{u_i, v_i, w_i\}$, $1 \le i \le k$, all three of the edges $\langle u_i, v_i\rangle$, $\langle u_i, w_i\rangle$, and $\langle v_i, w_i\rangle$ belong to E.
MEASURE: Cardinality of the triangle packing, i.e., the number of disjoint subsets V_i.

Good News: Approximable within $3/2+\varepsilon$ for any $\varepsilon > 0$ [Hurkens and Schrijver, 1989].
Bad News: APX-complete [Kann, 1991].
Comment: Transformation from bounded MAXIMUM 3-DIMENSIONAL MATCHING. Admits a PTAS for planar graphs [Baker, 1994] and for λ-precision unit disk graphs [Hunt III et al., 1994b]. Still APX-complete when the degree of G is bounded by 4 [Kann, 1991].
Garey and Johnson: GT11

GT11. MAXIMUM H-MATCHING

INSTANCE: Graph $G = \langle V_G, E_G\rangle$ and a fixed graph $H = \langle V_H, E_H\rangle$ with at least three vertices in some connected component.
SOLUTION: A H-matching for G, i.e., a collection $G_1, G_2, \ldots, G_k$ of disjoint subgraphs of G, each isomorphic to H.
MEASURE: Cardinality of the H-matching, i.e., the number of disjoint subgraphs G_i.

Good News: Approximable within $|V_H|/2+\varepsilon$ for any $\varepsilon > 0$ [Hurkens and Schrijver, 1989].
Bad News: APX-hard [Kann, 1994a].

Comment: Transformation from bounded MAXIMUM 3-SATISFIABILITY. Variation in which the degree of G is bounded by a constant B is APX-complete [Kann, 1994a]. Admits a PTAS for planar graphs [Baker, 1994], but does not admit an FPTAS [Berman, Johnson, Leighton, Shor and Snyder, 1990]. Admits a PTAS for λ-precision unit disk graphs [Hunt III et al., 1994b].
Induced MAXIMUM H-MATCHING, i.e., where the subgraphs G_i are induced subgraphs of G, has the same good and bad news as the ordinary problem, even when the degree of G is bounded.
MAXIMUM MATCHING OF CONSISTENT k-CLIQUES is the variation in which H is a k-clique and the vertices of G are partitioned into k independent sets $V_1, \dots, V_k$, each V_i is partitioned into some sets $V_{i,j}$, and at most one vertex in each $V_{i,j}$ may be included in the matching. This problem is not in APX for $k \geq 5$ and is not approximable within $4/3$ for $k \in \{2,3,4\}$. It is not in APX for any $k \geq 2$ unless NP$\subset$QP [Verbitsky, 1995].
Garey and Johnson: GT12

GT12. MINIMUM BOTTLENECK PATH MATCHING

INSTANCE: Graph $G = \langle V, E\rangle$ with an even number of vertices, and a weight function $w : E \to N$.
SOLUTION: A disjoint path perfect matching for G, i.e., a collection $P_1, P_2, \dots, P_{|V|/2}$ of disjoint simple paths in G with disjoint end points.
MEASURE: Weight of the heaviest path in the matching, i.e., $\max\limits_{1\leq i\leq |V|/2} \sum\limits_{e\in P_i} w(e)$.

Good News: Approximable within 2 [Datta and Sen, 1995].

GT13. MINIMUM CLIQUE PARTITION

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: A clique partition for G, i.e., a partition of V into disjoint subsets $V_1, V_2, \dots, V_k$ such that, for $1 \leq i \leq k$, the subgraph induced by V_i is a complete graph.
MEASURE: Cardinality of the clique partition, i.e., the number of disjoint subsets V_i.

Good News: Approximable within $O\left(|V|\frac{(\log\log|V|)^2}{(\log|V|)^3}\right)$ [Halldórsson, 1993a].
Bad News: Not approximable within $|V|^{\varepsilon}$ for some $\varepsilon > 0$ [Lund and Yannakakis, 1994].
Comment: Equivalent to MINIMUM GRAPH COLORING [Paz and Moran, 1981]. The complementary maximization problem, where $|V| - k$, is to be maximized, is approximable within $4/3$ [Halldórsson, 1996].
Garey and Johnson: GT15

GT14. MINIMUM k-CAPACITATED TREE PARTITION

INSTANCE: Graph $G = \langle V, E\rangle$, and a weight function $w : E \to N$.
SOLUTION: A k-capacitated tree partition of G, i.e., a collection of vertex disjoint subsets $E_1, \dots, E_m$ of E such that, for each i, the subgraph induced by E_i is a tree of at least k vertices.
MEASURE: The weight of the partition, i.e., $\sum_{e\in\bigcup_i E_i} w(e)$.

Good News: Approximable within $2 - 1/|V|$ [Goemans and Williamson, 1995a].

Comment: The variation in which the trees must contain exactly k vertices and the triangle inequality is satisfied is approximable within $4(1 - 1/k)(1 - 1/|V|)$. Similar results hold for the corresponding cycle and path partitioning problems with the triangle inequality [Goemans and Williamson, 1995a].

GT15. MINIMUM CLIQUE COVER

INSTANCE: Graph $G = \langle V, E\rangle$.

SOLUTION: A clique cover for G, i.e., a collection $V_1, V_2, \ldots, V_k$ of subsets of V, such that each V_i induces a complete subgraph of G and such that for each edge $\langle u, v\rangle \in E$ there is some V_i that contains both u and v.

MEASURE: Cardinality of the clique cover, i.e., the number of subsets V_i.

Good News: Approximable within $O(f(|V|))$ if MAXIMUM CLIQUE is approximable within $f(|V|)$ [Halldórsson, 1994].

Bad News: Not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Lund and Yannakakis, 1994].

Comment: Equivalent to MINIMUM CLIQUE PARTITION under ratio-preserving reduction [Kou, Stockmeyer, and Wong, 1978] and [Simon, 1990]. The complementary maximization problem, where $|E| - k$ is to be maximized, is approximable within $4/3$ [Halldórsson, 1996]. The constrained variation in which the input is extended with a positive integer k, a vertex $v_0 \in V$ and a subset S of V, and the problem is to find the clique cover of size k that contains the largest number of vertices from S, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].

Garey and Johnson: GT17

GT16. MINIMUM COMPLETE BIPARTITE SUBGRAPH COVER

INSTANCE: Graph $G = \langle V, E\rangle$.

SOLUTION: A complete bipartite subgraph cover for G, i.e., a collection $V_1, V_2, \ldots, V_k$ of subsets of V, such that each V_i induces a complete bipartite subgraph of G and such that for each edge $\langle u, v\rangle \in E$ there is some V_i that contains both u and v.

MEASURE: Cardinality of the complete bipartite subgraph cover, i.e., the number of subsets V_i.

Good News: Approximable within $O(f(|V|))$ if MAXIMUM CLIQUE is approximable within $f(|V|)$ [Halldórsson, 1994].

Bad News: Not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Lund and Yannakakis, 1994].

Comment: Equivalent to MINIMUM CLIQUE PARTITION under ratio-preserving reduction [Simon, 1990].

Garey and Johnson: GT18

GT17. MINIMUM VERTEX DISJOINT CYCLE COVER

INSTANCE: Graph $G = \langle V, E\rangle$.

SOLUTION: A family F of vertex disjoint cycles covering V.

MEASURE: Number of cycles in F.

Bad News: Not in Apx [Sahni and Gonzalez, 1976].
Comment: Variation in which the graph G is directed is not in Apx.

GT18. Minimum Edge Disjoint Cycle Cover

Instance: Graph $G = \langle V, E\rangle$.
Solution: A family F of edge disjoint cycles covering V.
Measure: Number of cycles in F.

Bad News: Not in Apx [Sahni and Gonzalez, 1976].
Comment: Variation in which the graph G is directed is not in Apx.

GT19. Minimum Cut Cover

Instance: Graph $G = \langle V, E\rangle$.
Solution: A collection $V_1, \ldots, V_m$ of cuts, i.e., a collection of subsets $V_i \subseteq V$ such that, for each edge $(u, v) \in E$, a subset V_i exists such that either $u \in V_i$ and $v \notin V_i$ or $u \notin V_i$ and $v \in V_i$.
Measure: Cardinality of the collection, i.e., m.

Good News: Approximable within $1+(\log|V|-3\log\log|V|)/opt(G)$ [Motwani and Naor, 1993].
Bad News: There is no polynomial-time algorithm with relative error less than 1.5. [Motwani and Naor, 1993].
Comment: The negative result is obtained by relating the problem with the coloring problem. Solvable in polynomial time for planar graphs. Observe that any graph has a cut cover of cardinality $\lceil \log|V| \rceil$.

Subgraphs and Supergraphs

GT20. Maximum Clique

Instance: Graph $G = \langle V, E\rangle$.
Solution: A clique in G, i.e. a subset $V' \subseteq V$ such that every two vertices in V' are joined by an edge in E.
Measure: Cardinality of the clique, i.e., $|V'|$.

Good News: Approximable within $O\left(|V|/(\log|V|)^2\right)$ [Boppana and Halldórsson, 1992].
Bad News: Not approximable within $|V|^{1/4-\varepsilon}$ for any $\varepsilon > 0$ [Bellare, Goldreich, and Sudan, 1995].
Comment: Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$, unless co-Rp=Np [Håstad, 1996]. The same problem as Maximum Independent Set on the complementary graph. Approximable within $opt/O(m^{3(k+1)})$ if $opt = |V|/k+m$ [Alon and Kahale, 1994] and [Mahajan and Ramesh, 1995]. The vertex weighted version is approximable within $O(|V|(\log\log|V|/\log|V|)^2)$ [Halldórsson, 1995b].
Garey and Johnson: GT19

GT21. MAXIMUM INDEPENDENT SET

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: An independent set of vertices, i.e. a subset $V' \subseteq V$ such that no two vertices in V' are joined by an edge in E.
MEASURE: Cardinality of the independent set, i.e., $|V'|$.

Good News: See MAXIMUM CLIQUE.
Bad News: See MAXIMUM CLIQUE.
Comment: The same problem as MAXIMUM CLIQUE on the complementary graph. Admits a PTAS for planar graphs [Baker, 1994] and for unit disk graphs [Hunt III et al., 1994b]. Variation in which the degree of G is bounded by a constant B for $B \geq 3$ is APX-complete [Papadimitriou and Yannakakis, 1991] and [Berman and Fujito, 1995], not approximable within B^{ε} for some $\varepsilon > 0$ [Alon, Feige, Wigderson, and Zuckerman, 1995], and is approximable within $(B+3)/5+\varepsilon$ for every $\varepsilon > 0$ [Berman and Fürer, 1994] and [Berman and Fujito, 1995]. For large values of B it is approximable within $O(B/\log\log B)$ [Halldórsson and Radhakrishnan, 1994]. Approximable within $k/2+\varepsilon$ for $k+1$-claw free graphs [Halldórsson, 1995a]. The vertex weighted version is approximable within $3/2$ for $B = 3$ [Hochbaum, 1983], and within $(B+2)/3$ for $B > 3$ [Halldórsson, 1995b].
MAXIMUM INDEPENDENT SET OF k-GONS, the variation in which the number of pairwise independent k-gons (cycles of size k, $k \geq 3$) is to be maximized and where two k-gons are independent if any edge connecting vertices from different k-gons must belong to at least one of these k-gons, is not approximable within $4/3$ for any $k \geq 3$. It is not in APX for any $k \geq g$ unless NP$\subset$QP [Verbitsky, 1995].
Garey and Johnson: GT20

GT22. MAXIMUM INDEPENDENT SEQUENCE

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: An independent sequence for G, i.e., a sequence $v_1, \dots, v_m$ of independent vertices of G such that, for all $i < m$, a vertex $\bar{v}_i \in V$ exists which is adjacent to v_{i+1} but is not adjacent to any v_j for $j \leq i$.
MEASURE: Length of the sequence, i.e., m.

Good News: Approximable within $O(|V|/\log|V|)$ [Halldórsson, 1995b].
Bad News: Not approximable within $|V|^{\varepsilon}$ for some $\varepsilon > 0$. [Blundo, De Santis, and Vaccaro, 1994].
Comment: Transformation from MAXIMUM CLIQUE.

GT23. MAXIMUM INDUCED SUBGRAPH WITH PROPERTY Π

INSTANCE: Graph $G = \langle V, E\rangle$. The property Π must be hereditary, i.e., every subgraph of G' satisfies Π whenever G' satisfies Π, and non-trivial, i.e., it is satisfied for infinitely many graphs and false for infinitely many graphs.
SOLUTION: A subset $V' \subseteq V$ such that the subgraph induced by V' has the property Π.
MEASURE: Cardinality of the induced subgraph, i.e., $|V'|$.

Good News: Approximable within $O(|V|/\log|V|)$ if Π is checkable in time 2^{n^c} for some constant c [Halldórsson, 1995b].

Bad News: Not approximable within $|V|^{\varepsilon}$ for some $\varepsilon > 0$ unless P=NP, if Π is false for some clique or independent set (for example planar, outerplanar, bipartite, complete bipartite, acyclic, degree-constrained, chordal, interval). Not approximable within $2^{\log^{0.5-\varepsilon}|V|}$ for any $\varepsilon > 0$ unless NP$\subset$QP, if Π is a non-trivial hereditary graph property (for example comparability, permutation, perfect, circular-arc, circle, line graph) [Lund and Yannakakis, 1993].

Comment: Approximable within $O(|V|(\log\log|V|/\log|V|)^2)$ if Π is false for some clique or independent set, and approximable within $(B+1)/3$ where B is the degree of the graph The positive results above are valid even for the vertex weighted problem [Halldórsson, 1995b]. The same problem on directed graphs is not approximable within $2^{\log^{0.5-\varepsilon}|V|}$ for any $\varepsilon > 0$ unless NP$\subset$QP, if Π is a non-trivial hereditary digraph property (for example acyclic, transitive, symmetric, antisymmetric, tournament, degree-constrained, line digraph) [Lund and Yannakakis, 1993]. Admits a PTAS for planar graphs if Π is hereditary and determined by the connected components, i.e., G' satisfies Π whenever every connected component of G' satisfies Π [Nishizeki and Chiba, 1988]. The good news is valid also for the vertex weighted version [Halldórsson, 1995b].

Garey and Johnson: GT21

GT24. Minimum Vertex Deletion to Obtain Subgraph with Property Π

Instance: Directed or undirected graph $G = \langle V, E\rangle$.
Solution: A subset $V' \subseteq V$ such that the subgraph induced by $V - V'$ has the property Π.
Measure: Cardinality of the set of deleted vertices, i.e., $|V'|$.

Good News: Approximable within some constant for any hereditary property Π with a finite number of minimal forbidden subgraphs (for example transitive digraph, symmetric, antisymmetric, tournament, line graph, and interval) [Lund and Yannakakis, 1993]. Approximable within some constant for any property Π that can be expressed as a universal first order sentence over subsets of edges of the graph [Kolaitis and Thakur, 1995].

Bad News: APX-hard for any non-trivial hereditary property Π [Lund and Yannakakis, 1993].

Comment: It is approximable within $O(\log|V|)$ if the subgraph has to be bipartite [Garg, Vazirani, and Yannakakis, 1994].

GT25. Minimum Edge Deletion to Obtain Subgraph with Property Π

Instance: Directed or undirected graph $G = \langle V, E\rangle$.
Solution: A subset $E' \subseteq E$ such that the subgraph $G = \langle V, E - E'\rangle$ has the property Π.
Measure: Cardinality of the set of deleted edges, i.e., $|E'|$.

Good News: Approximable within some constant for any property Π that can be expressed as a universal first order sentence over subsets of edges of the graph [Kolaitis and Thakur, 1995].

GT26. Maximum Induced Connected Subgraph with Property Π

Instance: Graph $G = \langle V, E\rangle$.
Solution: A subset $V' \subseteq V$ such that the subgraph induced by V' is connected and has the property Π.

MEASURE: Cardinality of the induced connected subgraph, i.e., $|V'|$.

Bad News: Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ if Π is a non-trivial hereditary graph property that is satisfied by all paths and is false for some complete bipartite graph (for example path, tree, planar, outerplanar, bipartite, chordal, interval) [Lund and Yannakakis, 1993].

Comment: NPOPB-complete when Π is either path or chordal [Berman and Schnitger, 1992]. NPOPB-complete and not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ when Π is simple cycle [Kann, 1995].

Garey and Johnson: GT22 and GT23

GT27. MINIMUM VERTEX DELETION TO OBTAIN CONNECTED SUBGRAPH WITH PROPERTY Π

INSTANCE: Directed or undirected graph $G = \langle V, E\rangle$.

SOLUTION: A subset $V' \subseteq V$ such that the subgraph induced by $V - V'$ is connected and has the property Π.

MEASURE: Cardinality of the set of deleted vertices, i.e., $|V'|$.

Bad News: Not approximable within $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ if Π is any non-trivial hereditary property determined by the blocks (for example planar, outerplanar, bipartite, chordal, cactus, acyclic graph, acyclic digraph, without cycles of specified length, symmetric digraph, antisymmetric digraph) [Yannakakis, 1979].

GT28. MAXIMUM DEGREE-BOUNDED CONNECTED SUBGRAPH

INSTANCE: Graph $G = \langle V, E\rangle$, a weight function $w : E \to N$, and an integer $d \geq 2$.

SOLUTION: A subset $E' \subseteq E$ such that the subgraph $G' = \langle V, E'\rangle$ is connected and has no vertex with degree exceeding d.

MEASURE: The total weight of the subgraph, i.e., $\sum_{e\in E'} w(E)$.

Bad News: Not in APX for any d [Kann, —].

Comment: Transformation from LONGEST PATH. The problems have, indeed, the same non-approximability results.

Garey and Johnson: GT26

GT29. MAXIMUM PLANAR SUBGRAPH

INSTANCE: Graph $G = \langle V, E\rangle$.

SOLUTION: A subset $E' \subseteq E$ such that $G' = \langle V, E'\rangle$ is planar.

MEASURE: Size of the subset, i.e., $|E'|$.

Good News: Approximable within 2.5 [Călinescu, Fernandes, Finkler, and Karloff, 1996].

Bad News: APX-complete [Călinescu, Fernandes, Finkler, and Karloff, 1996].

Comment: Transformation from MINIMUM METRIC TRAVELING SALESPERSON PROBLEM with distances one and two. The complementary problem *Minimum Nonplanar Edge Deletion* is APX-hard. Variation in which the subgraph should be outerplanar is APX-complete and approximable within 1.5 [Călinescu, Fernandes, Finkler, and Karloff, 1996].

Garey and Johnson: GT27

GT30. Minimum Edge Deletion k-partition

Instance: Graph $G = \langle V, E\rangle$ and a weight function $w : E \to N$.
Solution: An k-partition, i.e., a color assignment $c : V \to [1..k]$.
Measure: The weight of the monochromatic edges, i.e., $\sum_{(v_1,v_2)\in E:c(v_1)=c(v_2)} w(v_1, v_2)$.

Good News: Approximable within $\log |V|$ for $k = 2$ [Garg, Vazirani, and Yannakakis, 1996] and within $\varepsilon|V|^2$ for $k = 3$ and any $\varepsilon > 0$ [Kann, Khanna, Lagergren, and Panconesi, 1997].
Bad News: Apx-hard [Garg, Vazirani, and Yannakakis, 1996].
Comment: Not approximable within 1.058 for $k = 2$ [Håstad, 1997]. Not approximable within $O(|E|)$ for $k \geq 3$, even for graphs with $|E| = \Omega(|V|^{2-\varepsilon})$ for any $\varepsilon > 0$ [Kann, Khanna, Lagergren, and Panconesi, 1997]. Approximable within 9/4 for planar graphs and $k = 2$ [Goemans and Williamson, 1996].

GT31. Maximum k-Colorable Subgraph

Instance: Graph $G = \langle V, E\rangle$.
Solution: A subset $E' \subseteq E$ such that the subgraph $G' = \langle V, E'\rangle$ is k-colorable, i.e., there is a coloring for G' of cardinality at most k.
Measure: Cardinality of the subgraph, i.e., $|E'|$.

Good News: Approximable within $1/(1 - 1/k + 2 \ln k/k^2)$ [Frieze and Jerrum, 1995] and [Mahajan and Ramesh, 1995].
Bad News: Apx-complete for $k \geq 2$ [Papadimitriou and Yannakakis, 1991].
Comment: Equivalent to Maximum k-Cut for unweighted graphs. Admits a Ptas if $|E| = \Theta(|V|^2)$ and $k = o(|V|)$ [Arora, Karger, and Karpinski, 1995].

GT32. Maximum Edge Subgraph

Instance: Graph $G = \langle V, E\rangle$, a weight function $w : E \to N$, and positive integer k.
Solution: A subset $V' \subseteq V$ such that $|V'| = k$.
Measure: Total weight of the edges in the subgraph induced by V', i.e.,

$$\sum_{(u,v)\in E\cap(V'\times V')} w(u, v).$$

Good News: Approximable within $O(|V|^{1/3-\varepsilon})$ for some $\varepsilon > 0$ [Kortsarz and Peleg, 1993] and [Feige, Kortsarz, and Peleg, 1995].
Comment: Also called *Heaviest Subgraph*. Approximable within 2 if the weights satisfy the triangle inequality [Hassin, Rubinstein, and Tamir, 1994]. The unweighted problem admits a Ptas if $|E| = \Theta(|V|^2)$ and $k = \Theta(|V|)$ [Arora, Karger, and Karpinski, 1995].

GT33. Minimum Edge 2-Spanner

Instance: Connected graph $G = \langle V, E\rangle$.
Solution: A 2-spanner of G, i.e., a spanning subgraph G' of G such that, for any pair of vertices u and v, the shortest path between u and v in G' is at most twice the shortest path between u and v in G.

MEASURE: The number of edges in G'.

Good News: Approximable within $O(\log |V|)$ [Kortsarz and Peleg, 1992b].
Comment: The variation in which the goal is to minimize the maximum degree in G' is approximable within $O(\sqrt{\log |V|}\Delta^{1/4})$ where Δ is the maximum degree in G [Kortsarz and Peleg, 1994].

GT34. MAXIMUM k-COLORABLE INDUCED SUBGRAPH

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: A subset $V' \subseteq V$ such that the induced subgraph $G' = \langle V', E'\rangle$ is k-colorable, i.e., there is a coloring for G' of cardinality at most k.
MEASURE: Cardinality of the vertex set of the induced subgraph, i.e., $|V'|$.

Good News: As easy to approximate as MAXIMUM INDEPENDENT SET for $k \geq 1$ (finding k independent sets) [Halldórsson, 1994].
Bad News: As hard to approximate as MAXIMUM INDEPENDENT SET for $k \geq 1$ [Panconesi and Ranjan, 1993].
Comment: Transformation from MAXIMUM INDEPENDENT SET. Equivalent to MAXIMUM INDEPENDENT SET for $k = 1$. Admits a PTAS for 'δ-near-planar' instances for any $\delta \geq 0$ [Hunt III et al., 1994a]. Variation in which the degree of G is bounded by a constant B is approximable within $(B/k + 1)/2$ [Halldórsson, 1995a] and APX-complete.

GT35. MINIMUM EQUIVALENT DIGRAPH

INSTANCE: Directed graph $G = \langle V, E\rangle$.
SOLUTION: A subset $E' \subseteq E$ such that, for every ordered pair of vertices $u, v \in V$, the graph $G' = \langle V, E'\rangle$ contains a directed path from u to v if and only if G does.
MEASURE: Cardinality of E', i.e., $|E'|$.

Good News: Approximable within 1.645 [Khuller, Raghavachari, and Young, 1995].
Bad News: APX-complete [Khuller, Raghavachari, and Young, 1995].
Comment: APX-complete even if restricted to strongly connected graphs with no cycle longer than 17.
Garey and Johnson: GT33

GT36. MINIMUM INTERVAL GRAPH COMPLETION

INSTANCE: Graph $G = \langle V, E\rangle$.
SOLUTION: An interval graph $G' = \langle V, E'\rangle$ that contains G as a subgraph, i.e., $E \subseteq E'$. An interval graph is a graph whose vertices can be mapped to distinct intervals in the real line such that two vertices in the graph have an edge between them if and only if their corresponding intervals overlap.
MEASURE: The cardinality of the interval graph, i.e., $|E'|$.

Good News: Approximable within $O\left(\log^2 |V|\right)$ [Ravi, Agrawal, and Klein, 1991].
Garey and Johnson: GT35

GT37. Minimum Chordal Graph Completion

Instance: Graph $G = \langle V, E \rangle$.
Solution: A chordal graph completion, i.e., a superset E' containing E such that $G' = \langle V, E' \rangle$ is chordal, that is, for every simple cycle of more than 3 vertices in G', there is some edge in E' that is not involved in the cycle but that joins two vertices in the cycle.
Measure: The size of the completion, i.e., $|E' - E|$.

Good News: Approximable within $O(|E|^{1/4} \log^{3.5} |V|)$ [Klein, Agrawal, Ravi, and Rao, 1990].
Comment: Approximable within $O(\log^4 |V|)$ for graphs with bounded degree [Klein, Agrawal, Ravi, and Rao, 1990].
Garey and Johnson: OPEN4

GT38. Maximum Constrained Hamiltonian Circuit

Instance: Graph $G = \langle V, E \rangle$ and subset $S \subseteq E$ of the edges.
Solution: A Hamiltonian circuit C in G, i.e., a circuit that visits every vertex in V once.
Measure: Cardinality of the edges in S that are used in the circuit C, i.e., $|S \cap C|$.

Bad News: Not approximable within $|E|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Comment: Variation in which the graph is directed has the same bad news [Zuckerman, 1993].
Garey and Johnson: Similar to GT37 and GT38

Vertex Ordering

GT39. Minimum Bandwidth

Instance: Graph $G = \langle V, E \rangle$.
Solution: A linear ordering of V, i.e. a one-to-one function $f : V \to \{1, 2, \ldots, |V|\}$.
Measure: The bandwidth of the ordering, i.e. $\max_{(u,v) \in E} |f(u) - f(v)|$.

Good News: Approximable within $O(\log |V|)$ if G is a caterpillar [Haralambides, Makedon, and Monien, 1991]. . Approximable within 2 if G is asteroidal triple-free [Kloks, Kratsch, and Müller, 1996].
Garey and Johnson: GT40

GT40. Minimum Linear Arrangement

Instance: Graph $G = \langle V, E \rangle$.
Solution: A one-to-one function $f : V \to [1..|V|]$.
Measure: The total length of the edges in the linear arrangement, i.e. $\sum_{\{u,b\} \in E} |f(u) - f(v)|$.

Good News: Approximable within $O(\log^2 |V|)$ [Leighton and Rao, 1988].
Comment: Admits a Ptas if $|E| = \Theta(|V|^2)$. Variation in which the n points are on the d-dimensional grid instead of in a line and the rectilinear metric is used also admits a Ptas if $|E| = \Theta(|V|^2)$ [Arora, Frieze, and Kaplan, 1996].
Garey and Johnson: GT42

GT41. Minimum Cut Linear Arrangement

Instance: Graph $G = \langle V, E\rangle$.
Solution: A one-to-one function $f : V \to [1..|V|]$.
Measure: Maximum number of cut edges in any integer point, i.e.

$$\max_{i\in[1..|V|]} |\{\{u,v\} \in E : f(u) \le i < f(v)\}|.$$

Good News: Approximable within $O(\log^2 |V|)$ [Leighton and Rao, 1988].
Comment: Admits a Ptas if $|E| = \Theta(|V|^2)$ [Arora, Frieze, and Kaplan, 1996].
Garey and Johnson: GT44

Iso- and Other Morphisms

GT42. Maximum Common Subgraph

Instance: Graphs $G_1 = \langle V_1, E_1\rangle$ and $G_2 = \langle V_2, E_2\rangle$.
Solution: A common subgraph, i.e. subsets $E_1{}' \subseteq E_1$ and $E_2{}' \subseteq E_2$ such that the two subgraphs $G_1' = \langle V_1, E_1{}'\rangle$ and $G_2' = \langle V_2, E_2{}'\rangle$ are isomorphic.
Measure: Cardinality of the common subgraph, i.e., $|E'|$.

Bad News: Apx-hard [Halldórsson, 1994].
Comment: Transformation from Maximum Cut. Not approximable within an absolute error guarantee of $\min(|V_1|, |V_2|)^\varepsilon$ for some $\varepsilon > 0$ [Verbitsky, 1994]. Transformation to Maximum Clique with a quadratic vertex amplification [Kann, 1992b]. Variation in which the degree of the graphs G_1 and G_2 is bounded by the constant B is not harder to approximate than the bounded degree induced common subgraph problem and is approximable within $B+1$ [Kann, 1992a].
Garey and Johnson: GT49

GT43. Maximum Common Induced Subgraph

Instance: Graphs $G_1 = \langle V_1, E_1\rangle$ and $G_2 = \langle V_2, E_2\rangle$.
Solution: A common induced subgraph, i.e. subsets $V_1{}' \subseteq V_1$ and $V_2{}' \subseteq V_2$ such that $|V_1{}'| = |V_2{}'|$, and the subgraph of G_1 induced by $V_1{}'$ and the subgraph of G_2 induced by $V_2{}'$ are isomorphic.
Measure: Cardinality of the common induced subgraph, i.e., $|V_1{}'|$.

Bad News: Not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Kann, 1992a].
Comment: Transformations to and from Maximum Clique. Variation in which the degree of the graphs G_1 and G_2 is bounded by the constant B is Apx-hard and is approximable within $B+1$. If the induced subgraph is restricted to be connected the problem is Npopb-complete and not approximable within $(|V_1| + |V_2|)^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1992a].

GT44. Maximum Common Embedded Sub-tree

Instance: Trees T_1 and T_2 with labels on the nodes.
Solution: A common embedded sub-tree, i.e. a labeled tree T' that can be embedded into both T_1 and T_2. An embedding from T' to T is an injective function from the nodes of T' to the nodes of T that preserves labels and ancestorship. Note that since fathership does not need to be preserved, T' does not need to be an ordinary subtree.
Measure: Cardinality of the common embedded sub-tree, i.e., $|T'|$.

Good News: Approximable within $\log^2 n$, where n is the number of nodes in the trees [Halldórsson and Tanaka, 1996].
Bad News: Apx-hard [Zhang and Jiang, 1994].
Comment: Transformation from Maximum k-Set Packing. Variation in which the problem is to minimize the edit distance between the two trees is also Apx-hard.

GT45. Minimum Graph Transformation

Instance: Graphs $G_1 = \langle V_1, E_1 \rangle$ and $G_2 = \langle V_2, E_2 \rangle$.
Solution: A transformation that makes G_1 isomorphic to G_2. In the transformation a set of edges $E' \subseteq E_1$ is removed from E_1 and added to E_2.
Measure: Cardinality of the transformation set, i.e., $|E'|$.

Bad News: Apx-hard [Lin, 1994].
Comment: Transformation from Maximum 3-Satisfiability.

Miscellaneous

GT46. Longest Path with Forbidden Pairs

Instance: Graph $G = \langle V, E \rangle$ and a collection $C = \{\langle a_1, b_1 \rangle, \ldots, \langle a_m, b_m \rangle\}$ of pairs of vertices from V.
Solution: A simple path in G that contains at most one vertex from each pair in C.
Measure: Length of the path, i.e., the number of edges in the path.

Bad News: Npopb-complete [Berman and Schnitger, 1992].
Comment: Transformation from Longest Computation.
Garey and Johnson: GT54

GT47. Shortest Path with Forbidden Pairs

Instance: Graph $G = \langle V, E \rangle$, a collection $C = \{(a_1, b_1), \ldots, (a_m, b_m)\}$ of pairs of vertices from V, an initial vertex $s \in V$, and a final vertex $f \in V$.
Solution: A simple path from s to f in G that contains at most one vertex from each pair in C.
Measure: Length of the path, i.e., the number of edges in the path.

Bad News: Npopb-complete [Kann, 1994b].
Comment: Transformation from Shortest Computation.

GT48. Minimum Point-To-Point Connection

Instance: Graph $G = \langle V, E \rangle$, a weight function $w : E \to N$, a set $S = \{s_1, \dots, s_p\}$ of sources, and a set $D = \{d_1, \dots, d_p\}$ of destinations.
Solution: A point-to-point connection, i.e., a subset $E' \subseteq E$ such that each source-destination pair is connected in E'.
Measure: The weight of the connection, i.e., $\sum_{e \in E'} w(e)$.

Good News: Approximable within $2 - 1/p$ [Goemans and Williamson, 1995a].

GT49. Minimum Metric Dimension

Instance: Graph $G = \langle V, E \rangle$.
Solution: A metric basis for G, i.e., a subset $V' \subseteq V$ such that for each pair $u, v \in V$ there is a $w \in V'$ such that the length of the shortest path from u to w is different from the length of the shortest path from v to w.
Measure: The cardinality of the metric basis, i.e., $|V'|$.

Good News: Approximable within $O(\log |V|)$ [Khuller, Raghavachari, and Rosenfeld, 1994].
Garey and Johnson: GT61

GT50. Minimum Tree Width

Instance: Graph $G = \langle V, E \rangle$.
Solution: A tree decomposition, i.e., a pair $\langle \{X_i : i \in I\}, T \rangle$ where $T = \langle I, F \rangle$ is a tree and $\{X_i\}$ is a collection of subsets of V, such that
1. $\bigcup_{i \in I} X_i = V$,
2. for any $(v, w) \in E$, there exists an $i \in I$ with $u, v \in X_i$,
3. for any $v \in V$, the set $\{i \in I : v \in X_i\}$ forms a connected subtree of T.

Measure: The tree width of the tree decomposition, i.e., $\max_{i \in I} |X_i| - 1$.

Good News: Approximable within $O(\log |V|)$ [Bodlaender et al., 1995].
Bad News: There is no polynomial-time algorithm with an absolute error guarantee of $|V|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Bodlaender et al., 1995].
Comment: The *Minimum Path Width*, the variation in which T is a path, is approximable within $O(\log^2 |V|)$ and has the same negative bound. Similar problems with the same positive and negative results are *Minimum Elimination Tree Height* and *Minimum Front Size* [Bodlaender et al., 1995].

GT51. Minimum Graph Inference

Instance: Class C of undirected edgecolored graphs, string x of colors.
Solution: A graph $G \in |C|$ and a simple path in G such that the string of colors traversed in the path is equal to x.
Measure: Cardinality of the edge set of G.

Bad News: Apx-hard and not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Maruyama and Miyano, 1995].

Comment: The same negative results are valid also if all graphs in C are caterpillars.

Network Design

Spanning Trees

ND1. Minimum k-Spanning Tree

Instance: Graph $G = \langle V, E \rangle$, an integer $k \leq n$, and a weight function $w : E \to N$.
Solution: A k-spanning tree, i.e., a subtree T of G of at least k nodes.
Measure: The weight of the tree, i.e., $\sum_{e \in T} w(e)$.

Good News: Approximable within 3 [Garg, 1996].
Comment: The restriction to points in the Euclidean plane admits a Ptas [Arora, 1996]. The analogous diameter and communication-cost k-spanning tree problems are not in Apx [Ravi, Sundaram, Marathe, Rosenkrantz, and Ravi, 1994].

ND2. Minimum Degree Spanning Tree

Instance: Graph $G = \langle V, E \rangle$.
Solution: A spanning tree for G.
Measure: The maximum degree of the spanning graph.

Good News: Approximable with an absolute error guarantee of 1 [Fürer and Raghavachari, 1994].
Bad News: Not approximable within $3/2 - \varepsilon$ for any $\varepsilon > 0$ [Garey and Johnson, 1979].
Comment: Apx-intermediate unless the polynomial-hierarchy collapses [Crescenzi, Kann, Silvestri, and Trevisan]. The generalization of the problem to Steiner trees is also approximable with an absolute error guarantee of 1 [Fürer and Raghavachari, 1994].
Garey and Johnson: ND1

ND3. Minimum Geometric 3-Degree Spanning Tree

Instance: Set $P \subseteq Z \times Z$ of points in the plane.
Solution: A spanning tree T for P in which no vertex has degree larger than 3.
Measure: The total weight of the spanning tree, i.e., $\sum_{(u,v) \in T} d(u, v)$, where $d(u, v)$ is the Euclidean distance between u and v.

Good News: Admits a Ptas [Arora, 1996].
Comment: The 4-degree spanning tree problem also admits a Ptas, but the Np-hardness of the problem is open [Arora, 1996]. The 5-degree problem is polynomial-time solvable. In d-dimensional Euclidean space for $d \geq 3$ the 3-degree spanning tree problem is approximable within 5/3. [Khuller, Raghavachari, and Young, 1996a].

ND4. Maximum Leaf Spanning Tree

Instance: Graph $G = \langle V, E \rangle$.
Solution: A spanning tree for G.

MEASURE: The number of leaves of the spanning tree.

Good News: Approximable within 3 [Lu and Ravi, 1992].
Bad News: APX-complete [Galbiati, Maffioli, and Morzenti, 1994].
Comment: Other problems which aim at finding spanning trees that maximize a single objective function have been considered. In particular, the problems of finding a spanning tree that has maximum diameter, or maximum height with respect to a specified root are not in APX, the problems of finding a spanning tree that has maximum sum of the distances between all pairs of vertices, or maximum sum of the distances from a specified root, are not in PTAS, while the problem of finding a spanning tree that maximizes the number of paths which connect pairs of vertices and pass through a common arc is approximable within 9/8 [Galbiati, Maffioli, and Morzenti, 1995].
Garey and Johnson: ND2

ND5. MAXIMUM MINIMUM METRIC k-SPANNING TREE

INSTANCE: Graph $G = \langle V, E\rangle$, length $l(e) \in N$ for each $e \in E$ satisfying the triangle inequality.
SOLUTION: A subset $V' \subseteq V$ such that $|V'| = k$.
MEASURE: Cost of the minimum spanning tree of the subgraph induced by V'.

Good News: Approximable within 4 [Halldórsson, Iwano, Katoh, and Tokuyama, 1995].
Bad News: APX-complete and not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Halldórsson, Iwano, Katoh, and Tokuyama, 1995].
Comment: Also called *Maximum Remote Minimum Spanning Tree.* Reduction from MAXIMUM INDEPENDENT SET. As hard to approximate as MAXIMUM INDEPENDENT SET for non-metric graphs. Approximable within 2.252 in the Euclidean metric, but not known to be NP-complete. The *Maximum Minimum k-Steiner Tree* problem is approximable within 3 and not approximable within $4/3-\varepsilon$, but approximable within 2.16 in the Euclidean metric. *Maximum Minimum Metric k-TSP* is approximable within 3 and not approximable within $2-\varepsilon$.

ND6. MINIMUM DIAMETER SPANNING SUBGRAPH

INSTANCE: Graph $G = \langle V, E\rangle$, weight $w(e) \in Z^+$ and length $l(e) \in N$ for each $e \in E$, positive integer B.
SOLUTION: A spanning subgraph $E' \subseteq E$ for G such that the sum of the weights of the edges in E' does not exceed B.
MEASURE: The diameter of the spanning subgraph.

Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Plesník, 1981].
Comment: Not approximable within $3/2-\varepsilon$ for planar graphs. Not approximable within $5/4-\varepsilon$ if $l(e) = 1$ for every edge e [Plesník, 1981]. For results on minimum Steiner trees of bounded diameter, see MINIMUM STEINER TREE.
Garey and Johnson: Similar to ND4

ND7. MINIMUM STEINER TREE

INSTANCE: Complete graph $G = \langle V, E\rangle$, a metric given by edge weights $s : E \to N$ and a subset $S \subseteq V$ of required vertices.

SOLUTION: A Steiner tree, i.e., a subtree of G that includes all the vertices in S.
MEASURE: The sum of the weights of the edges in the subtree.

Good News: Approximable within 1.644 [Karpinski and Zelikovsky, 1995].
Bad News: APX-complete [Bern and Plassmann, 1989].
Comment: Variation in which the weights are only 1 or 2 is still APX-complete, but approximable within 4/3 [Bern and Plassmann, 1989]. The geometric Steiner tree problem, where the vertices lie in the plane and the Euclidean metric is used, admits a PTAS [Arora, 1996], even if the vertices lie in a Euclidean space of constant dimension d [Arora, 1997]. When all weights lie in an interval $[\alpha, \alpha(1+1/k)]$ the problem is approximable within $1+1/ek+O(1/k^2)$ [Halldórsson, Ueno, Nakao, and Kajitani, 1992]. The variation in which the diameter of the tree is bounded by a constant d the problem is not approximable within $(1-\varepsilon)\ln|V|$ for any $\varepsilon > 0$, unless NP$\subset$ DTIME$(n^{\log\log n})$, but approximable within $\log|V|$ for $d \in \{4,5\}$ [Bar-ilan, Kortsarz, and Peleg, 1996] and within $d\log|V|$ for any constant $d \geq 6$ [Kortsarz and Peleg, 1997].
A prize-collecting variation in which a penalty is associated with each vertex and the goal is to minimize the cost of the tree and the vertices in S not in the tree is approximable within $2-1/(|V|-1)$ [Goemans and Williamson, 1995a]. Another prize-collecting variation in which a salesperson has to sell some quota R while traveling the least distance possible is approximable within $O(\log^2(\min(R,|V|)))$ [Awerbuch, Azar, Blum, and Vempala, 1995]. The variation in which an integer $k \leq |S|$ is given in input and at least k vertices of S must be included in the subtree is approximable within $4\sqrt{k}$ [Ravi, Sundaram, Marathe, Rosenkrantz, and Ravi, 1994]. Variation in which there are groups of required vertices and each group must be touched by the Steiner tree is approximable within $g-1$, where g is the number of groups [Ihler, 1991]; when the number of groups is unbounded the problem is harder than MINIMUM DOMINATING SET to approximate, even if all edge weights are 1 [Ihler, 1992].
The constrained variation in which the input is extended with a positive integer k and a subset T of E, and the problem is to find the Steiner tree of weight at most k that contains the largest number of edges from T, is not approximable within $|E|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993]. If the solution is allowed to be a forest with at most q trees, for a given constant q, the problem is approximable within $2(1-1/(|S|-q+1))$ [Ravi, 1994a]. Finally, if the topology of the Steiner tree is given as input, the problem admits a PTAS [Jiang, Lawler, and Wang, 1994].
Garey and Johnson: ND12

ND8. MINIMUM GEOMETRIC STEINER TREE

INSTANCE: Set $P \subseteq Z \times Z$ of points in the plane.
SOLUTION: A finite set of Steiner points, i.e., $Q \subseteq Z \times Z$.
MEASURE: The total weight of the minimum spanning tree for the vertex set $P \cup Q$, where the weight of an edge $\langle (x_1,y_1),(x_2,y_2)\rangle$ is the discretized Euclidean length

$$\left\lceil \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \right\rceil .$$

Good News: Approximable within 1.144 [Zelikovsky, 1994].

Comment: Approximable within 1.26 in the rectilinear metric [Berman and Ramaiyer, 1994] and [Karpinski and Zelikovsky, 1995]. Admits a PTAS if the input is c-local for some constant $c > 0$, where c-local means that in the minimum spanning tree the longest edge is at least c times as long as the shortest [Jiang and Wang, 1994]. *Minimum Steiner Trees with Obstacles*, the problem where a polygonally bounded region R is given in the input, and the Steiner tree has to lie inside R, admits a FPTAS under some restrictions of the input [Provan, 1988]. Variation of the rectilinear metric problem in which there are groups of required vertices and each group must be touched by the Steiner tree is APX-hard, even if the groups are defined by non-overlapping intervals on one of two parallel lines [Ihler, 1991].
Garey and Johnson: ND13

ND9. MINIMUM GENERALIZED STEINER NETWORK

INSTANCE: Graph $G = \langle V, E\rangle$, a weight function $w : E \to N$, a capacity function $c : E \to N$, and a requirement function $r : V \times V \to N$.
SOLUTION: A Steiner network over G that satisfies all the requirements and obeys all the capacities, i.e., a function $f : E \to N$ such that, for each edge e, $f(e) \leq c(e)$ and, for any pair of nodes i and j, the number of edge disjoint paths between i and j is at least $r(i, j)$ where, for each edge e, $f(e)$ copies of e are available.
MEASURE: The cost of the network, i.e., $\sum_{e\in E} w(e)f(e)$.

Good News: Approximable within $2 \cdot \mathcal{H}(R)$ where R is the maximum requirement and, for any n, $\mathcal{H}(n) = \sum_{i=1}^{n} \frac{1}{i}$ [Goemans, Goldberg, Plotkin, Shmoys, Tardos, and Williamson, 1994].
Comment: Also called *Minimum Survivable Network*. When all the requirements are equal, it is approximable within 2 [Khuller and Vishkin, 1994]. The variation in which there are no capacity constraints on the edges is approximable within $2 \cdot \mathcal{H}(|V|)$ [Aggarwal and Garg, 1994]. This problem belongs to the class of problems of finding a minimum-cost subgraph such that the number of edges crossing each cut is at least a specified requirement, which is a function f of the cut. If f is symmetric and maximal then any problem in this class is approximable within $2R$, where R is the maximum value of f over all cuts [Williamson, Goemans, Mihail, and Vazirani, 1995].

ND10. MINIMUM ROUTING TREE CONGESTION

INSTANCE: Graph $G = \langle V, E\rangle$ and a weight function $w : E \to N$.
SOLUTION: A routing tree T for G, i.e., a tree T in which each internal vertex has degree 3 and the leaves correspond to vertices of G.
MEASURE: The congestion of the routing tree, i.e., the maximum, for any edge e, of

$$\sum_{(u,v)\in E, u\in S, v\notin S} w(u, v)$$

where S is one of the two connected components obtained by deleting e from T.

Good News: Approximable within $\log |V|$ [Khuller, Raghavachari, and Young, 1993].
Bad News: Not in APX [Seymour and Thomas, 1994].
Comment: The algorithm extends to the case when the routing tree is allowed to have vertices of higher degree. If G is planar [Seymour and Thomas, 1994], or if T is required to be a spanning tree and G is complete [Khuller, Raghavachari, and Young, 1993], the problem is solvable in polynomial time.

Cuts and Connectivity

ND11. Maximum Cut

Instance: Graph $G = \langle V, E \rangle$.
Solution: A partition of V into disjoint sets V_1 and V_2.
Measure: The cardinality of the cut, i.e., the number of edges with one end point in V_1 and one endpoint in V_2.

Good News: Approximable within 1.1383 [Goemans and Williamson, 1995b].
Bad News: Apx-complete [Papadimitriou and Yannakakis, 1991].
Comment: Not approximable within 1.0624 [Håstad, 1997]. Admits a Ptas if $|E| = \Theta(|V|^2)$ [Arora, Karger, and Karpinski, 1995]. Variation in which the degree of G is bounded by a constant B for $B \geq 3$ is still Apx-complete [Papadimitriou and Yannakakis, 1991] and [Alimonti and Kann, 1997]. The weighted problem, where every edge is assigned a nonnegative weight and the objective is to maximize the total weight of the edges in the cut is also approximable within 1.1383 [Goemans and Williamson, 1995b]. *Maximum Bisection*, the weighted problem with the additional constraint that the partition must cut the graph into halves of the same size, is approximable within 1.54 [Frieze and Jerrum, 1995] and [Mahajan and Ramesh, 1995]. Variation in which some pairs of vertices are restricted to be on the same side (or on different sides) is still approximable within 1.138 [Goemans and Williamson, 1995b].
Garey and Johnson: ND16

ND12. Minimum Crossing Number

Instance: Directed graph $G = \langle V, A \rangle$.
Solution: An embedding of G in the plane.
Measure: The number of pairs of edges crossing one another.

Good News: Approximable within 3 in the case of bipartite graphs [Eades and Wormald, 1994].

Garey and Johnson: OPEN3

ND13. Maximum Directed Cut

Instance: Directed graph $G = \langle V, A \rangle$.
Solution: A partition of V into disjoint sets V_1 and V_2.
Measure: The cardinality of the cut, i.e., the number of arcs with one end point in V_1 and one endpoint in V_2.

Good News: Approximable within 1.165 [Feige and Goemans, 1995].
Bad News: Apx-complete [Papadimitriou and Yannakakis, 1991].
Comment: Not approximable within 1.083 [Håstad, 1997]. Admits a Ptas if $|E| = \Theta(|V|^2)$ [Arora, Karger, and Karpinski, 1995]. The weighted problem, where every arc is assigned a nonnegative weight and the objective is to maximize the total weight of the arcs in the cut is also approximable within 1.165 [Feige and Goemans, 1995].

ND14. MAXIMUM k-CUT

INSTANCE: Graph $G = \langle V, E \rangle$, a weight function $w : E \to N$, and an integer $k \in [2..|V|]$.
SOLUTION: A partition of V into k disjoint sets $F = \{C_1, C_2, \ldots, C_k\}$.
MEASURE: The sum of the weight of the edges between the disjoint sets, i.e.,

$$\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{\substack{v_1 \in C_i \\ v_2 \in C_j}} w(\{v_1, v_2\}).$$

Good News: Approximable within $1/(1 - 1/k + 2\ln k/k^2)$ [Frieze and Jerrum, 1995] and [Mahajan and Ramesh, 1995].
Bad News: APX-complete.
Comment: The unweighted version is equivalent to MAXIMUM k-COLORABLE SUBGRAPH. Approximable within 1.21 for $k = 3$, 1.18 for $k = 4$, and 1.15 for $k = 5$ [Frieze and Jerrum, 1995]. Not approximable within $1 + 1/(34k)$ [Kann, Khanna, Lagergren, and Panconesi, 1997]. The constrained variation in which the input is extended with a positive integer W, a vertex $v_0 \in V$ and a subset S of V, and the problem is to find the 2-cut of weight at least W with the largest number of vertices from S on the same side as v_0, is not approximable within $|V|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].

ND15. MINIMUM NETWORK INHIBITION ON PLANAR GRAPHS

INSTANCE: Planar graph $G = \langle V, E \rangle$, capacity function $c : E \to N$, destruction cost function $d : E \to N$, and budget B.
SOLUTION: An attack strategy to the network, i.e., a function $\alpha : E \to [0, 1]$ such that $\sum_{e \in E} \alpha(e)d(e) \le B$.
MEASURE: The capability left in the damaged network, i.e., the minimum cut in G with capacity c' defined as $c'(e) = \alpha(e)c(e)$.

Good News: Admits an FPTAS [Phillips, 1993].

ND16. MINIMUM k-CUT

INSTANCE: Graph $G = \langle V, E \rangle$, a weight function $w : E \to N$, and an integer $k \in [2..|V|]$.
SOLUTION: A partition of V into k disjoint sets $F = \{C_1, C_2, \ldots, C_k\}$.
MEASURE: The sum of the weight of the edges between the disjoint sets, i.e.,

$$\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{\substack{v_1 \in C_i \\ v_2 \in C_j}} w(\{v_1, v_2\}).$$

Good News: Approximable within $2 - \frac{2}{k}$ [Saran and Vazirani, 1991].
Comment: Solvable in polynomial time $O(|V|^{k^2})$ for fixed k [Goldschmidt and Hochbaum, 1988]. If the sets in the partition are restricted to be of equal size, the problem is approximable within $|V| \cdot (k - 1)/k$ [Saran and Vazirani, 1991]. The unweighted problem admits a PTAS if every vertex has degree $\Theta(|V|)$ [Arora, Karger, and Karpinski, 1995].

ND17. Minimum Vertex k-Cut

Instance: Graph $G = \langle V, E\rangle$, a set $S = \{s_1, t_1, \ldots, s_k, t_k\}$ of special vertices, and a weight function $w : V - S \to N$, and an integer k.
Solution: A vertex k-cut, i.e., a subset $C \subseteq V - S$ of vertices such that their deletion from G disconnects each s_i from t_i for $1 \le i \le k$.
Measure: The sum of the weight of the vertices in the cut, i.e., $\sum_{v \in C} w(v)$.

Good News: Approximable within $O(\log |V|)$ [Garg, Vazirani, and Yannakakis, 1994].

ND18. Minimum Multiway Cut

Instance: A graph $G = \langle V, E\rangle$, a set $S \subseteq V$ of terminals, and a weight function $w : E \to N$.
Solution: A multiway cut, i.e., a set $E' \subseteq E$ such that the removal of E' from E disconnects each terminal from all the others.
Measure: The weight of the cut, i.e., $\sum_{e \in E'} w(e)$.

Good News: Approximable within $2 - \frac{2}{|S|}$ [Dahlhaus, Johnson, Papadimitriou, Seymour, and Yannakakis, 1994].
Bad News: Apx-complete [Dahlhaus, Johnson, Papadimitriou, Seymour, and Yannakakis, 1994].
Comment: Admits a Ptas if every vertex has degree $\Theta(|V|)$ [Arora, Karger, and Karpinski, 1995]. It remains Apx-complete even if $|S| \ge 3$ is fixed. For $|S| = 4$ and $|S| = 8$ it is approximable within $4/3$ and $12/7$, respectively. In the case of directed graphs the problem is approximable within $O(\log |S|)$ and Apx-hard [Garg, Vazirani, and Yannakakis, 1994]. The vertex deletion variation is approximable within $2 - 2/|S|$ and is Apx-complete [Garg, Vazirani, and Yannakakis, 1994].

ND19. Minimum Multi-Cut

Instance: A graph $G = \langle V, E\rangle$, a set $S \subseteq V \times V$ of source-terminal pairs, and a weight function $w : E \to N$.
Solution: A multi-cut, i.e., a set $E' \subseteq E$ such that the removal of E' from E disconnects s_i from t_i for every pair $(s_i, t_i) \in S$.
Measure: The weight of the cut, i.e., $\sum_{e \in E'} w(e)$.

Good News: Approximable within $O(\log |S|)$ [Garg, Vazirani, and Yannakakis, 1996]
Bad News: Apx-hard.
Comment: Generalization of Minimum Multiway Cut. When the graph is a tree the problem is approximable within 2 and is Apx-complete even for trees of height one and unit edge weight [Garg, Vazirani, and Yannakakis, 1993]. The variation in which vertices instead of edges should be deleted is also approximable within $O(\log |S|)$ [Garg, Vazirani, and Yannakakis, 1994]. The variation in which the input specifies a set of demand edges and a given node v^* and the solution must be a feasible v^*-cut, i.e., the demand edges must either be in the cut or have both endpoints on the opposite part of v^* is approximable within 2.8 [Yu and Cheriyan, 1995].

ND20. Minimum Ratio-Cut

Instance: Graph $G = \langle V, E\rangle$, a capacity function $c : E \to N$, and k commodities, i.e., k pairs $(s_i, t_i) \in V^2$ and a demand d_i for each pair.
Solution: A cut, i.e., a partition of V into two disjoint sets V_1 and V_2.
Measure: The capacity of the cut divided by the demand across the cut, i.e.,

$$\sum_{\substack{v_1 \in V_1, v_2 \in V_2 \\ (v_1, v_2) \in E}} c(v_1, v_2) / \sum_{i : |\{s_i, t_i\} \cap V_1| = 1} d_i.$$

Good News: Approximable within $O(\log k)$ [Aumann and Rabaini, 1996].
Comment: Also called *Sparsest Cut.* In the uniform-demand case the problem is in Apx [Klein, Plotkin, and Rao, 1993].

ND21. Minimum b-Balanced Cut

Instance: Graph $G = \langle V, E\rangle$, a vertex-weight function $w : V \to N$, an edge-cost function $c : E \to N$, and a rational b, $0 < b \le 1/2$.
Solution: A cut C, i.e., a subset $C \subseteq V$, such that

$$\min\{w(C), w(V - C)\} \ge b \cdot w(V),$$

where $w(V')$ denotes the sum of the weights of the vertices in V'.
Measure: The weight of the cut, i.e.,

$$\sum_{e \in \delta(C)} c(e)$$

where $\delta(C) = \{e = \{v_1, v_2\} : e \in E, v_1 \in C, v_2 \in V - C\}$.

Bad News: Not approximable within $1 + |V|^{2-\varepsilon}/opt$ for any $\varepsilon > 0$ [Bui and Jones, 1992].
Comment: Also called *Minimum b-Edge Separator.* There is a polynomial algorithm that finds a b-balanced cut within an $O(\log |V|)$ factor of the optimal $b + \varepsilon$-balanced cut for $b \le 1/3$ [Shmoys, 1997]. Approximable within 2 for planar graphs for $b \le 1/3$ if the vertex weights are polynomially bounded [Garg, Saran, and Vazirani, 1994]. Not approximable within $1 + |V|^{1/2-\varepsilon}/opt$ for graphs of maximum degree 3 [Bui and Jones, 1992]. The unweighted problem admits a Ptas if every vertex has degree $\Theta(|V|)$ for $b \le 1/3$ [Arora, Karger, and Karpinski, 1995]. *Minimum k-Multiway Separator*, the variation in which the removal of the cut edges partitions the graph into at most k parts, where the sum of the vertex weights in each part is at most $2\sum_{v\in V} w(v)/k$, is approximable within $(2 + o(1)) \ln |V|$ [Even, Naor, Rao, and Schieber, 1997].

ND22. Minimum b-Vertex Separator

Instance: Graph $G = \langle V, E\rangle$ and a rational b, $0 < b \le 1/2$.
Solution: A partition of V into disjoint sets A, B, and C, such that $\max\{|A|, |B|\} \le b \cdot |V|$ and no edge has one endpoint in A and one in B.
Measure: The size of the separator, i.e., $|C|$.

Bad News: Not approximable within $1 + |V|^{1/2-\varepsilon}/opt$ for any $\varepsilon > 0$, even for graphs of maximum degree 3. [Bui and Jones, 1992].

Comment: For planar graphs a separator of size $O(\sqrt{|V|})$ can be found in polynomial time [Lipton and Tarjan, 1979]. An $f(|V|)$-approximation algorithm is also a $f(16|V|^5)$ algorithm for Minimum b-Balanced Cut[Bui and Jones, 1992].

ND23. Minimum Quotient Cut

Instance: Graph $G = \langle V, E \rangle$, a vertex-weight function $w : V \to N$, and an edge-cost function $c : E \to N$.

Solution: A cut C, i.e., a subset $C \subseteq V$.

Measure: The quotient of the cut, i.e.,

$$\frac{c(C)}{\min\{w(C), w(V-C)\}}$$

where $c(C)$ denotes the sum of the costs of the edges (u, v) such that either $u \in C$ and $v \notin C$ or $u \notin C$ and $v \in C$ and, for any subset $V' \subseteq V$, $w(V')$ denotes the sum of the weights of the vertices in V'.

Good News: Approximable within $O(\log |V|)$ [Leighton and Rao, 1988].

Comment: Also called *Minimum Flux Cut.* Admits a Ptas for planar graphs [Park and Phillips, 1993]. The generalization to hypergraphs, also called *Minimum Net Expansion,* is approximable within $O(\log |E|)$ in the uniform vertex-weight case [Makedon and Tragoudas, 1990]. Other approximation algorithms for hypergraph partitioning problems are contained in [Makedon and Tragoudas, 1990].

ND24. Minimum k-Vertex Connected Subgraph

Instance: Graph $G = \langle V, E \rangle$. k is a constant, $k \geq 2$.

Solution: A k-vertex connected spanning subgraph $G' = \langle V, E' \rangle$ for G, i.e. a spanning subgraph that cannot be disconnected by removing fewer than k vertices.

Measure: The cardinality of the spanning subgraph, i.e., $|E'|$.

Good News: Approximable within $1+1/k$ [Garg, Santosh, and Singla, 1993] and [Cheriyan and Thurimella, 1996].

Comment: On directed graphs the problem is approximable within 1.61 for $k = 1$ [Khuller, Raghavachari, and Young, 1996b], and within $1 + 1/k$ for $k \geq 2$ [Cheriyan and Thurimella, 1996]. Variation in which each edge has a nonnegative weight and the objective is to minimize the total weight of the spanning subgraph is approximable within $2+1/|V|$ for $k = 2$ [Khuller and Raghavachari, 1995] and within $2\sum_{i=1}^{k} \frac{1}{i}$ for $k > 2$ [Ravi and Williamson, 1995]. If the weight function satisfies the triangle inequality, the problem is approximable within $3/2$ for $k = 2$ [Frederickson and Jájá, 1982] and within $2 + 2(k-1)/|V|$ for $k > 2$ [Khuller and Raghavachari, 1995].

Garey and Johnson: GT31

ND25. Minimum k-Edge Connected Subgraph

Instance: Graph $G = \langle V, E \rangle$. k is a constant, $k \geq 2$.

SOLUTION: A k-edge connected spanning subgraph $G' = \langle V, E' \rangle$ for G, i.e. a spanning subgraph that cannot be disconnected by removing fewer than k edges.
MEASURE: The cardinality of the spanning subgraph, i.e., $|E'|$.

Good News: Approximable within 1.704 [Khuller and Raghavachari, 1995 and Cheriyan and Thurimella, 1996 and Fernandes, 1997].
Bad News: APX-complete even for $k = 2$ [Fernandes, 1997].
Comment: Approximable within $\min\{1 + 5/k, 7/4 - (3k^2 - 6k + 8)/(2k^2(3k - 2))\}$ for even k and within $\min\{1 + 7/k, 7/4 - (2k - 3)/(4k^2)\}$ for odd k [Cheriyan and Thurimella, 1996 and Fernandes, 1997]. On directed graphs the problem is approximable within 1.61 for $k = 1$ [Khuller, Raghavachari, and Young, 1996b], within 2 for $2 \le k \le 16$, and within $1 + 4/\sqrt{k}$ for $k \ge 17$ [Cheriyan and Thurimella, 1996]. Variation in which each edge has a nonnegative weight and the objective is to minimize the total weight of the spanning subgraph is approximable within 2 for every k [Khuller and Vishkin, 1994].

ND26. MINIMUM BICONNECTIVITY AUGMENTATION

INSTANCE: Graph $G = \langle V, E \rangle$ and a symmetric weight function $w : V \times V \to N$.
SOLUTION: A connectivity augmenting set E' for G, i.e., a set E' of unordered pairs of vertices from V such that $G' = \langle V, E \cup E' \rangle$ is biconnected.
MEASURE: The weight of the augmenting set, i.e., $\sum_{(u,v) \in E'} w(u, v)$.

Good News: Approximable within 2 [Frederickson and Jájá, 1981] and [Khuller and Thurimella, 1993].
Comment: The same bound is valid also when G' must be bridge connected (edge connected) [Khuller and Thurimella, 1993]. Minimum k-Connectivity Augmentation, the problem in which G' has to be k-connected (vertex or edge connected), is also approximable within 2 [Khuller, 1997]. If the weight function satisfies the triangle inequality, the problem is approximable within 3/2 [Frederickson and Jájá, 1982]. If $E = \emptyset$ the problem is the same as weighted MINIMUM k-VERTEX CONNECTED SUBGRAPH.
Garey and Johnson: ND18

ND27. MINIMUM STRONG CONNECTIVITY AUGMENTATION

INSTANCE: Directed graph $G = \langle V, A \rangle$ and a weight function $w : V \times V \to N$.
SOLUTION: A connectivity augmenting set A' for G, i.e., a set A' of ordered pairs of vertices from V such that $G' = \langle V, A \cup A' \rangle$ is strongly connected.
MEASURE: The weight of the augmenting set, i.e., $\sum_{(u,v) \in E'} w(u, v)$.

Good News: Approximable within 2 [Frederickson and Jájá, 1981].
Comment: The unweighted problem (i.e. where all weights are 1) is approximable within 1.61 [Khuller, Raghavachari, and Young, 1996b].
Garey and Johnson: ND19

ND28. MINIMUM BOUNDED DIAMETER AUGMENTATION

INSTANCE: Graph $G = \langle V, E \rangle$, positive integer $D < |V|$.

SOLUTION: An augmenting set E' for G, i.e., a set E' of unordered pairs of vertices from V, such that $G' = \langle V, E \cup E' \rangle$ has diameter D, i.e., the maximum distance of any pair of vertices is at most D.
MEASURE: Cardinality of the augmenting set, i.e., $|E'|$.

Bad News: As hard to approximate as MINIMUM SET COVER [Li, McCormick, and Simchi-Levi, 1992].
Comment: Variation in which the size of the augmenting set is bounded by D and the problem is to minimize the diameter is approximable within $4 + 2/opt$ [Li, McCormick, and Simchi-Levi, 1992].

Routing Problems

ND29. MINIMUM TRAVELING SALESPERSON

INSTANCE: Set C of m cities, distances $d(c_i, c_j) \in N$ for each pair of cities $c_i, c_j \in C$.
SOLUTION: A tour of C, i.e., a permutation $\pi : [1..m] \to [1..m]$.
MEASURE: The length of the tour, i.e., $d\left(\{c_{\pi(m)}, c_{\pi(1)}\}\right) + \sum_{i=1}^{m-1} d\left(\{c_{\pi(i)}, c_{\pi(i+1)}\}\right)$.

Bad News: NPO-complete [Orponen and Mannila, 1987].
Comment: The corresponding maximization problem (finding the tour of maximum length) is approximable within 7/5 if the distance function is symmetric and 63/38 if it is asymmetric [Kosaraju, Park, and Stein, 1994].
Garey and Johnson: ND22

ND30. MINIMUM METRIC TRAVELING SALESPERSON PROBLEM

INSTANCE: Set C of m cities, distances $d(c_i, c_j) \in N$ satisfying the triangle inequality.
SOLUTION: A tour of C, i.e., a permutation $\pi : [1..m] \to [1..m]$.
MEASURE: The length of the tour.

Good News: Approximable within 3/2 [Christofides, 1976].
Bad News: APX-complete [Papadimitriou and Yannakakis, 1993].
Comment: Approximable within $\log m$ if the distance function is asymmetric [Frieze, Galbiati, and Maffioli, 1982]. Variation in which the distances are only 1 or 2 is still APX-complete, but approximable within 7/6 [Papadimitriou and Yannakakis, 1993] if the distance function is symmetric and 17/12 if it is asymmetric [Vishwanathan, 1992]. In the special case in which the distances are the shortest path lengths in a given unweighted maximal planar graph, a solution can be found whose length is at most $3/2(|V|-3)$ [Nishizeki, Asano, and Watanabe, 1983]. The generalization in which, for each city, a neighborhood is specified in which the salesperson can meet the client, is also approximable for a variety of neighborhood types such as unit segments, unit circles, and unit rectangles [Arkin and Hassin, 1993]. Another generalization in which the salesperson has to rearrange some objects while following the route is approximable within 2.5 [Anily and Hassin, 1992]. A prize-collecting variation in which a penalty is associated with each vertex and the goal is to minimize the cost of the tour and the vertices not in the tour is approximable within $2 - 1/(|V| - 1)$ [Goemans and Williamson, 1995a]. A clustered generalization in which vertices are partitioned into clusters

that must be traversed consecutively is approximable within 7/2 [Arkin, Hassin, and Klein, 1994]. A variation in which vertices can be revisited and the goal is to minimize the sum of the latencies of all vertices, where the latency of a vertex c is the length of the tour from the starting point to c, is approximable within 29 and is APX-complete [Blum, Chalasani, Coppersmith, Pulleyblank, Raghavan, and Sudan, 1994]. A combination of this problem and the matching problem, also called *Printed Circuit Board Assembly*, is approximable within 2.5 [Michel, Schroeter, and Srivastav, 1995].

ND31. MINIMUM GEOMETRIC TRAVELING SALESPERSON

INSTANCE: Set $C \subseteq Z \times Z$ of m points in the plane.
SOLUTION: A tour of C, i.e., a permutation $\pi : [1..m] \to [1..m]$.
MEASURE: The length of the tour, where the distance between (x_1, y_1) and (x_2, y_2) is the discretized Euclidean length

$$\left\lceil \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \right\rceil .$$

Good News: Admits a PTAS [Arora, 1996].
Comment: Generalization to d dimensions for d constant also admits a PTAS [Arora, 1997]. In R^m the problem is APX-complete for any l_p metric [Trevisan, 1997]. The variation in which an integer $k \leq m$ is given in the input and only at least k of the cities must be included in the tour also admits a PTAS [Arora, 1996]. Minimum geometric angular TSP, the variation in which the sum of the direction changes in the tour is minimized, is approximable within $O(\log m)$. The same bound is valid also when there may be several tours covering all the cities [Aggarwal, Coppersmith, Khanna, Motwani, and Schieber, 1997]. The maximum geometric traveling salesperson problem (finding the tour of maximum length) admits a nonconstructive PTAS, i.e., the algorithm does not produce the approximate tour [Barvinok, 1996].
Garey and Johnson: ND23

ND32. MINIMUM METRIC TRAVELING k-SALESPERSON PROBLEM

INSTANCE: Set C of m cities, an initial city $s \in C$, distances $d(c_i, c_j) \in N$ satisfying the triangle inequality.
SOLUTION: A collection of k subtours, each containing the initial city s, such that each city is in at least one subtour.
MEASURE: The maximum length of the k subtours.

Good News: Approximable within $1 - 1/k$ plus the performance ratio of the MINIMUM METRIC TRAVELING SALESPERSON PROBLEM, i.e., within $\frac{5}{2} - \frac{1}{k}$ [Frederickson, Hecht, and Kim, 1978].

ND33. MINIMUM METRIC BOTTLENECK WANDERING SALESPERSON PROBLEM

INSTANCE: Set C of m cities, an initial city $s \in C$, a final city $f \in C$, distances $d(c_i, c_j) \in N$ satisfying the triangle inequality.

SOLUTION: A simple path from the initial city s to the final city f passing through all cities in C, i.e., a permutation $\pi : [1..m] \to [1..m]$ such that $v_{\pi(1)} = s$ and $v_{\pi(m)} = f$.
MEASURE: The length of the largest distance in the path, i.e.,

$$\max_{i \in [1..m-1]} d\left(\{c_{\pi(i)}, c_{\pi(i+1)}\}\right).$$

Good News: Approximable within 2 [Hochbaum and Shmoys, 1986].
Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].
Comment: The same positive and negative results hold even if X is a set of point in d-dimensional space with the L_1 or L_∞ metric. If the L_2 metric is used then the upper bound is 1.969 [Feder and Greene, 1988]. The corresponding maximization problem called *Maximum Scatter TSP*, where the length of the shortest distance in the path is maximized, is approximable within 2, but not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Arkin, Chiang, Mitchell, Skiena, and Yang, 1997].
Garey and Johnson: ND24

ND34. MINIMUM CHINESE POSTMAN FOR MIXED GRAPHS

INSTANCE: Mixed graph $G = \langle V, A, E\rangle$, length $l(e) \in N$ for each $e \in A \cup E$.
SOLUTION: A cycle in G (possibly containing repeated vertices) that includes each directed and undirected edge at least once, traversing directed edges only in the specified direction.
MEASURE: The total length of the cycle.

Good News: Approximable within 5/3 [Frederickson, 1979].
Comment: Approximable within 3/2 for planar graphs [Frederickson, 1979].
Garey and Johnson: ND25

ND35. MINIMUM k-CHINESE POSTMAN PROBLEM

INSTANCE: Multigraph $G = \langle V, E\rangle$, initial vertex $s \in V$, length $l(e) \in N$ for each $e \in E$.
SOLUTION: A collection of k cycles, each containing the initial vertex s, that collectively traverse every edge in the graph at least once.
MEASURE: The maximum length of the k cycles.

Good News: Approximable within $2 - 1/k$ [Frederickson, Hecht, and Kim, 1978].

ND36. MINIMUM STACKER CRANE PROBLEM

INSTANCE: Mixed graph $G = \langle V, A, E\rangle$, length $l(e) \in N$ for each $e \in A \cup E$ such that for every arc there is a parallel edge of no greater length.
SOLUTION: A cycle in G (possibly containing repeated vertices) that includes each directed edge in A at least once, traversing such edges only in the specified direction.
MEASURE: The total length of the cycle.

Good News: Approximable within 9/5 [Frederickson, Hecht, and Kim, 1978].
Garey and Johnson: ND26

ND37. Minimum k-Stacker Crane Problem

Instance: Mixed graph $G = \langle V, A, E\rangle$, initial vertex $s \in V$, length $l(e) \in N$ for each $e \in A \cup E$,
Solution: A collection of k cycles, each containing the initial vertex s, that collectively traverse each directed edge in A at least once.
Measure: The maximum length of the k cycles.

Good News: Approximable within $14/5 - 1/k$ [Frederickson, Hecht, and Kim, 1978].

ND38. Minimum General Routing

Instance: Graph $G = \langle V, E\rangle$, length $l(e) \in N$ for each $e \in E$, subset $E' \subseteq E$, subset $V' \subseteq V$.
Solution: A cycle in G that visits each vertex in V' exactly once and traverses each edge in E'.
Measure: The total length of the cycle.

Good News: Approximable within 3/2 [Jansen, 1992].
Comment: The special case where $V' = V$ is called the rural postman problem.
Garey and Johnson: Generalization of ND27

ND39. Longest Path

Instance: Graph $G = \langle V, E\rangle$.
Solution: Simple path in G, i.e., a sequence of distinct vertices $v_1, v_2, \ldots, v_m$ such that, for any $1 \le i \le m-1$, $(v_i, v_{i+1}) \in E$.
Measure: Length of the path, i.e., the number of edges in the path.

Good News: Approximable within $O(|V|/\log|V|)$ [Alon, Yuster, and Zwick, 1994].
Bad News: Not in Apx [Karger, Motwani, and Ramkumar, 1993].
Comment: Transformation from Minimum Metric Traveling Salesperson Problem with distances one and two: Apx-hard and self-improvable. Not approximable within $2^{\log^{1-\varepsilon}|V|}$ for any $\varepsilon > 0$ unless Np⊂Qp [Karger, Motwani, and Ramkumar, 1993]. Similar results hold for a chromatic version of the problem [Bellare, 1993]. Variation in which the path must be induced subgraph of G, Longest Induced Path, is Npopb-complete and not approximable within $O(|V|^{1-\varepsilon})$ for any $\varepsilon > 0$, see Maximum Induced Connected Subgraph with Property Π.
Garey and Johnson: ND29

ND40. Shortest Weight-Constrained Path

Instance: Graph $G = \langle V, E\rangle$, length function $l : E \to N$, weight function $w : E \to N$, specified vertices $s, t \in V$, and integer W.
Solution: A simple path in G with total weight at most W, i.e., a sequence of distinct vertices $s = v_1, v_2, \ldots, v_m = t$ such that, for any $1 \le i \le m-1$, $(v_i, v_{i+1}) \in E$ and $\sum_{i=1}^{m_1} w(v_i, v_{i+1}) \le W$.
Measure: The length of the path, i.e., $\sum_{i=1}^{m_1} l(v_i, v_{i+1})$.

Good News: Admits an Fptas [Hassin, 1992] and [Phillips, 1993].
Garey and Johnson: ND30

ND41. MINIMUM RECTILINEAR GLOBAL ROUTING

INSTANCE: $m \times n$-array of gates, collection C of nets, i.e., 3-sets of gates.
SOLUTION: Wires following rectilinear paths connecting the gates in each net.
MEASURE: The largest number of wires in the same channel between two gates in the array.

Good News: Admits a PTAS$^\infty$ if $opt \in \omega(\ln(mn))$ [Raghavan and Thompson, 1991].
Comment: Approximable within $1 + (e-1)\sqrt{2\ln(mn)/opt}$ if $opt > 2\ln(mn)$. In APX if $opt \in \Omega(\ln(mn))$. The approximation algorithm will work also for nets with more than three gates, but the running time is exponential in the number of terminals.

Flow Problems

ND42. MAXIMUM PRIORITY FLOW

INSTANCE: Directed graph $G = \langle V, E \rangle$, sources $s_1, \ldots, s_k \in V$, sinks $t_1, \ldots, t_k \in V$, a capacity function $c : E \to R$, a bound function $b : V \to R$, and, for any vertex v, a partial order on the set of edges leaving v.
SOLUTION: A priority flow f, i.e., a function $f : E \to R$ such that (a) for any edge e, $f(e) \le c(e)$, (b) for any vertex $v \in V - \{s_1, \ldots, s_k, t_1, \ldots, t_k\}$, the flow is conserved at v, (c) for any vertex v, the flow leaving v is at most $b(v)$, and (d) for any vertex v and for any pair of edges e_1, e_2 leaving v, if $f(e_1) < c(e_1)$ and e_1 is less than e_2, then $f(e_2) = 0$.
MEASURE: The amount of flow entering sink t_1, i.e., $\sum_{(x,t_1)\in E} f(x, t_1)$.

Bad News: Not approximable within $2^{\theta(\log^p n)}$ for some $p > 0$ unless NP$\subseteq$ DTIME$(n^{d \log^{1/\varepsilon} n})$ [Bellare, 1993].
Comment: Does not admit a PTAS.

ND43. MAXIMUM INTEGRAL k-MULTICOMMODITY FLOW ON TREES

INSTANCE: A tree $T = \langle V, E \rangle$, a capacity function $c : E \to N$ and k pairs of vertices (s_i, t_i).
SOLUTION: A flow f_i for each pair (s_i, t_i) with $f_i \in N$ such that, for each $e \in E$, $\sum_{i=1}^k f_i q_i(e) \le c(e)$ where $q_i(e) = 1$ if e belongs to the unique path from s_i and t_i, 0 otherwise.
MEASURE: The sum of the flows, i.e., $\sum_{i=1}^k f_i$.

Good News: Approximable within 2 [Garg, Vazirani, and Yannakakis, 1993].
Bad News: APX-complete [Garg, Vazirani, and Yannakakis, 1993].
Comment: Transformation from MAXIMUM 3-DIMENSIONAL MATCHING. It remains APX-complete even if the edge capacities are 1 and 2.

ND44. MAXIMUM DISJOINT CONNECTING PATHS

INSTANCE: Graph $G = \langle V, E \rangle$, collection of vertex pairs $T = \{(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)\}$.
SOLUTION: A collection of edge disjoint paths in G connecting some of the pairs (s_i, t_i), i.e. sequences of vertices $u_1, u_2, \ldots, u_m$ such that, for some i, $u_1 = s_i$, $u_m = t_i$, and for any j, $(u_j, u_{j+1}) \in E$.
MEASURE: The number of vertex pairs (s_i, t_i) that are connected by the paths.

Comment: Approximable within 2 if G is a tree with parallel edges. Approximable within $O(\log|V|)$ if G is a two-dimensional mesh [Aumann and Rabani, 1995], or more generally, if G is a nearly-Eulerian uniformly high-diameter planar graph [Kleinberg and Tardos, 1995].

Minimum Path Coloring is the variation in which each path is assigned a color, where only paths with the same color need to be edge disjoint, and where the objective is to minimize the number of colors that are needed to connect all vertex pairs in the input. This problem is approximable within 3/2 on trees, within 2 on cycles [Raghavan and Upfal, 1994], within $O(\text{polylog}\log|V|)$ on two-dimensional meshes [Rabani, 1996], and within $O(\log|V|)$ on nearly-Eulerian uniformly high-diameter planar graphs [Kleinberg and Tardos, 1995].

Garey and Johnson: Similar to ND40

ND45. Minimum Maximum Disjoint Connecting Paths

Instance: Graph $G = \langle V, E\rangle$, length function $l : E \to N$, and a pair of vertices s, t in V.

Solution: Two vertex disjoint paths in G connecting s and t, i.e. two sequences of vertices $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ such that $|\{u_1, u_2, \ldots, u_m, v_1, v_2, \ldots, v_n\}| = m + n$, (s, u_1), (s, v_1), (u_m, t), and (v_n, t) are included in E, and for any i, (u_i, u_{i+1}) and (v_i, v_{i+1}) are included in E.

Measure: The longest of the two paths, i.e. $\max(l(s,u_1)+\sum_{i=1}^{m-1} l(u_i,u_{i+1})+l(u_m,t), l(s,v_1)+\sum_{i=1}^{n-1} l(v_i,v_{i+1}) + l(v_n,t))$.

Good News: Approximable within 2 [Li, McCormick, and Simchi-Levi, 1990].

Comment: Approximable within 2 also if the paths should be *edge disjoint* instead of vertex disjoint. Variation in which the graph is directed and we look for two vertex (edge) disjoint directed paths is also approximable within 2, and is not approximable within $2 - \varepsilon$ for any $\varepsilon > 0$ [Li, McCormick, and Simchi-Levi, 1990].

Garey and Johnson: Similar to ND41

ND46. Minimum Single-Sink Edge Installation

Instance: Graph $G = \langle V, E\rangle$, length function $l : E \to N$, set of sources $S \subseteq V$, sink $t \in V$, demand function $d : S \to Z^+$, finite set of cable types where each cable type is specified by its capacity and its cost per unit length.

Solution: A network of cables in the graph, consisting of an integral number of each cable type for each edge in G, that routes all the demands at the sources to the sink. The demand of each source must follow a single path from source to sink.

Measure: The total cost of building the network of cables.

Comment: Approximable within $O(\log D)$, where $D = \sum_{s\in S} d(s)$, for points in the Euclidean plane. Restricted version where the network to be designed must be a two-level tree (so that every path from a source to the sink consists of at most two edges) is approximable within $O(\log|V|)$ [Salman, Cheriyan, Ravi, and Subramanian, 1997].

Miscellaneous

ND47. MINIMUM BROADCAST TIME

INSTANCE: Graph $G = \langle V, E\rangle$ and a source node $v_0 \in V$.
SOLUTION: A broadcasting scheme. At time 0 only v_0 contains the message that is to be broadcast to every vertex. At each time step any vertex that has received the message is allowed to communicate the message to at most one of its neighbours.
MEASURE: The broadcast time, i.e., the time when all vertices have received the message.

Good News: Approximable within $O(\log^2 |V| / \log\log |V|)$ [Ravi, 1994b].
Comment: Approximable within $2B$ if the degree of G is bounded by a constant B [Ravi, 1994b]. Approximable within $O(\log |V|)$ if G is chordal, k-outerplanar [Kortsarz and Peleg, 1992a]. Approximable within $O(\log |V| / \log\log |V|)$ if G has bounded tree width [Marathe et al., 1995].
Minimum Gossip Time, the extension in which there is a message on every node and the objective is to minimize the time until every node has received every message, is approximable within twice the performance ratio of MINIMUM BROADCAST TIME [Ravi, 1994b].
Garey and Johnson: ND49

ND48. MINIMUM k-CENTER

INSTANCE: Complete graph $G = \langle V, E\rangle$ and distances $d(v_i, v_j) \in N$ satisfying the triangle inequality.
SOLUTION: A k-center set, i.e., a subset $C \subseteq V$ with $|C| = k$.
MEASURE: The maximum distance from a vertex to its nearest center, i.e., $\max_{v\in V} \min_{c\in C} d(v, c)$.

Good News: Approximable within 2 [Hochbaum and Shmoys, 1986].
Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Hsu and Nemhauser, 1979] and [Plesník, 1980].
Comment: Not in APX if the distances do not satisfy the triangle inequality [Hochbaum, 1997].
Minimum Capacitated k-Center, the variation in which the number of vertices each center can serve is bounded by a constant L, is approximable within 5 [Khuller and Sussmann, 1996]. The converse problem, where the maximum distance from each vertex to its center is given and the number of centers is to be minimized, is approximable within $\log L + 1$ [Bar-Ilan and Peleg, 1991]. The geometric k-center problem, where the vertices lie in the plane and the geometric metric is used, is approximable within 2, but is not approximable within 1.822 [Feder and Greene, 1988]. Variants of the geometric k-center problem are also studied in the paper. The rectilinear k-center problem, where the vertices lie in the plane and the L_∞ metric is used, is approximable within 2, but is not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Ko, Lee, and Chang, 1990]. The vertex weighted version, where the objective is to minimize the maximum weighted distance $d(v, c)w(v)$, is approximable within 2 [Plesník, 1987]. It is not approximable within $2-\varepsilon$ even if the distances are induced by a planar graph of maximum degree 3 with edge lengths 1 and vertex weights 1 [Plesník, 1980].
Minimum Absolute k-Center, the variation in which we allow the points in C to lie in edges (considered as curves) is also approximable within 2 and is not approximable within $2-\varepsilon$ [Plesník, 1988]. *Minimum α-All-Neighbor k-Center*, the variation in which we want to minimize the distance from each vertex to α of the k centers, is approximable within 2 when $\alpha \in \{2, 3\}$ and within 3 otherwise (even for the vertex weighted version) [Khuller, Pless, and Sussmann, 1997]. The asymmetric k-center problem, where $d(v_i, v_j)$ might be different from $d(v_j, v_i)$, is approximable within $O(\log^* |V|)$ [Vishwanathan, 1996].

Garey and Johnson: Similar to ND50

ND49. MINIMUM k-CLUSTERING

INSTANCE: Finite set X, a distance $d(x,y) \in N$ for each pair $x,y \in X$. The distances must satisfy the triangle inequality.
SOLUTION: A partition of X into disjoint subsets $C_1, C_2, \ldots, C_k$.
MEASURE: The largest distance between two elements in the same subset, i.e., $\max_{\substack{i\in[1..k]\\ x,y\in C_i}} d(x,y)$.

Good News: Approximable within 2 [Gonzalez, 1985] and [Hochbaum and Shmoys, 1986].
Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Gonzalez, 1985] and [Hochbaum and Shmoys, 1986].
Comment: The same positive and negative results are valid for the geometric (Euclidean) k-clustering problem in 3 dimensions [Gonzalez, 1985] and for rectilinear k-clustering problem in 2 dimensions. The geometric k-clustering problem in 2 dimensions is not approximable within 1.969 [Feder and Greene, 1988]. Other variants are also studied in this paper.
Garey and Johnson: MS9

ND50. MINIMUM k-CLUSTERING SUM

INSTANCE: Finite set X, a distance $d(x,y) \in N$ for each pair $x,y \in X$.
SOLUTION: A partition of X into disjoint subsets $C_1, C_2, \ldots, C_k$.
MEASURE: The sum of all distances between elements in the same subset, i.e.,

$$\sum_{i=1}^{k} \sum_{v_1,v_2\in C_i} d(v_1,v_2).$$

Bad News: Not in APX [Sahni and Gonzalez, 1976].
Comment: Approximable within 1.7 for $p=2$ and within 2 for $p \geq 3$ if d satisfies the triangle inequality [Guttman and Hassin, 1996].

ND51. MINIMUM k-SUPPLIER

INSTANCE: Complete graph $G = \langle V,E\rangle$, distances $d(v_i,v_j) \in N$ satisfying the triangle inequality, center construction cost $c(v) \in N$ and usage weight $w(v)$ for each $v \in V$, cost bound $L \in N$.
SOLUTION: A supplier set of legal cost, i.e., a subset $S \subseteq V$ such that $\sum_{v\in S} c(v) \leq L$.
MEASURE: The maximum weighted distance from a vertex to its nearest supplier, i.e.,

$$\max_{v\in V} \min_{s\in S} w(v)d(v,s).$$

Good News: Approximable within 3 [Wang and Cheng, 1990].
Bad News: Not approximable within $3-\varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].

ND52. Minimum k-Median

Instance: Complete graph $G = \langle V, E \rangle$ and distances $d(e) \in N$.
Solution: A k-median set, i.e., a subset $V' \subseteq V$ with $|V'| = k$.
Measure: The sum of the distances from each vertex to its nearest median, i.e.,

$$\sum_{v \in V} \min_{w \in V'} d(v, w).$$

Bad News: Not in Apx [Lin and Vitter, 1992].
Comment: The problem is in Apx if a small violation of the cardinality of the median set is allowed.
Garey and Johnson: ND51

ND53. Minimum Diameters Decomposition

Instance: Graph $G = \langle V, E \rangle$.
Solution: A decomposition of the graph into two factors, F_1 and F_2, with equal diameters.
Measure: The diameter om F_1.

Bad News: Not approximable within $3/2-\varepsilon$ for any $\varepsilon > 0$ [Plesník, 1982].
Comment: Variation in which the diameters of F_1 and F_2 do not need to be equal, and where the objective is to minimize the sum of the diameters, is not approximable within $5/4-\varepsilon$ [Plesník, 1982].

ND54. Maximum k-Facility Dispersion

Instance: Complete graph $G = \langle V, E \rangle$ and distances $d(v_i, v_j) \in N$ satisfying the triangle inequality.
Solution: A set of k facilities, i.e., a subset $F \subseteq V$ with $|F| = k$.
Measure: The minimum distance between two facilities, i.e., $\min_{f_1, f_2 \in F} d(f_1, f_2)$.

Good News: Approximable within 2 [Ravi, Rosenkrantz, and Tayi, 1991].
Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Ravi, Rosenkrantz, and Tayi, 1991].
Comment: Not in Apx if the distances do not satisfy the triangle inequality. *Maximum Edge Subgraph* is the variation where the measure is the average distance between any pair of facilities [Ravi, Rosenkrantz, and Tayi, 1991]. Variation in which we allow the points in F to lie in edges (considered as curves) is also approximable within 2 and is not approximable within $3/2 - \varepsilon$ [Tamir, 1991].

ND55. Maximum k-Facility Location

Instance: Complete graph $G = \langle V, E \rangle$ and profits $p(v_i, v_j) \in N$.
Solution: A set of k facilities, i.e., a subset $F \subseteq V$ with $|F| = k$.
Measure: The maximum total profit, i.e., $\sum_{v \in V} \max_{f \in F} p(v, f)$.

Good News: Approximable within $e/(e-1)$ [Cornuejols, Fisher, and Nemhauser, 1977].

ND56. Minimum k-Switching Network

Instance: Complete graph $G = \langle V, E\rangle$ and distances $d(v_i, v_j) \in N$ satisfying the triangle inequality.
Solution: A partition $F = \{A_1, A_2, \ldots, A_k, B_1, B_2, \ldots, B_k\}$ of V.
Measure: Maximum distance between vertices in different sets with the same index, i.e.,

$$\max_{i\in[1..k]} \max_{\substack{v_1\in A_i \\ v_2\in B_i}} d(v_1, v_2).$$

Good News: Approximable within 3 [Hochbaum and Shmoys, 1986].
Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Hochbaum and Shmoys, 1986].

ND57. Minimum Bend Number

Instance: Directed planar graph $G = \langle V, E\rangle$.
Solution: A planar orthogonal drawing of G, i.e., a drawing mapping vertices of G into points in the plane and edges of G into chains of horizontal and vertical segments such that no two edges cross.
Measure: Number of bends in the drawing.

Bad News: Not approximable within $1 + \frac{|V|^{1-\varepsilon}}{opt(G)}$ for any $\varepsilon > 0$ [Garg and Tamassia, 1994].

ND58. Minimum Length Triangulation

Instance: Collection $C = \{(a_i, b_i) : 1 \le i \le n\}$ of pairs of integers giving the coordinates of n points in the plane.
Solution: A triangulation of the set of points represented by C, i.e., a collection E of non-intersecting line segments each joining two points in C that divides the interior of the convex hull into triangular regions.
Measure: The discrete-Euclidean length of the triangulation, i.e.,

$$\left\lceil \sum_{((a_i,b_i),(a_j,b_j))\in E} \sqrt{(a_i - a_j)^2 + (b_i - b_j)^2} \right\rceil .$$

Good News: Approximable within $24 \log n$ [Plaisted and Hong, 1987].
Comment: Note that the problem is not known to be Np-complete. For a convex point set the problem is approximable within 12 [Plaisted and Hong, 1987]. The Steiner variation in which the point set of E must be a superset of C is approximable within 316 [Eppstein, 1992].
Garey and Johnson: OPEN12

ND59. Minimum Separating Subdivision

Instance: A family of disjoint polygons $P_1, \ldots, P_k$.
Solution: A separating subdivision, i.e., a family of k polygons $R_1, \ldots, R_k$ with pairwise disjoint boundaries such that, for each i, $P_i \subseteq R_i$.

MEASURE: The size of the subdivision, i.e., the total number of edges of the polygons $R_1, \ldots, R_k$.

Good News: Approximable within 7 [Mitchell and Suri, 1992].

Comment: The problem of separating a family of three-dimensional convex polyhedra is approximable within $O(\log n)$ while the problem of separating two d-dimensional convex polyhedra is approximable within $O(d \log n)$ where n denotes the number of facets in the input family.

Sets and Partitions

Covering, Hitting, and Splitting

SP1. MAXIMUM 3-DIMENSIONAL MATCHING

INSTANCE: Set $T \subseteq X \times Y \times Z$, where X, Y, and Z are disjoint.

SOLUTION: A matching for T, i.e., a subset $M \subseteq T$ such that no elements in M agree in any coordinate.

MEASURE: Cardinality of the matching, i.e., $|M|$.

Good News: Approximable within $3/2+\varepsilon$ for any $\varepsilon > 0$ [Hurkens and Schrijver, 1989].

Bad News: APX-complete [Kann, 1991].

Comment: Transformation from MAXIMUM 3-SATISFIABILITY. Admits a PTAS for 'planar' instances [Nishizeki and Chiba, 1988]. Variation in which the number of occurrences of any element in X, Y or Z is bounded by a constant B is APX-complete for $B \geq 3$ [Kann, 1991]. The generalized Maximum k-Dimensional Matching problem is approximable within $k/2+\varepsilon$ for any $\varepsilon > 0$ [Halldórsson, 1994]. The constrained variation in which the input is extended with a subset S of T, and the problem is to find the 3-dimensional matching that contains the largest number of elements from S, is not approximable within $|T|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993].

Garey and Johnson: SP1

SP2. MAXIMUM SET PACKING

INSTANCE: Collection C of finite sets.

SOLUTION: A set packing, i.e., a collection of disjoint sets $C' \subseteq C$.

MEASURE: Cardinality of the set packing, i.e., $|C'|$.

Good News: See MAXIMUM CLIQUE.

Bad News: See MAXIMUM CLIQUE.

Comment: Also called *Maximum Hypergraph Matching.* Equivalent to MAXIMUM CLIQUE under PTAS-reduction, where $|C| = |V|$ [Ausiello, D'Atri, and Protasi, 1980]. Therefore approximation algorithms and nonapproximability results for MAXIMUM CLIQUE will carry over to MAXIMUM SET PACKING. The problem MAXIMUM k-SET PACKING, the variation in which the cardinality of all sets in C are bounded from above by a constant $k \geq 3$, is APX-complete [Kann, 1991], and is approximable within $k/2+\varepsilon$ for any $\varepsilon > 0$ [Hurkens and Schrijver, 1989]. Still APX-complete when the number of occurrences in C of any element is bounded by a constant B for $B \geq 2$ Berman and Fujito, 1995]. Approximable within B if the number of occurrences in C of any element is bounded by B, even for the weighted variation of the problem [Hochbaum, 1983].

Garey and Johnson: SP3

SP3. MAXIMUM SET SPLITTING

INSTANCE: Collection C of subsets of a finite set S.
SOLUTION: A partition of S into two disjoint subsets S_1 and S_2.
MEASURE: Cardinality of the subsets in C that are not entirely contained in either S_1 or S_2.

Good News: Approximable within 2.
Bad News: APX-complete [Petrank, 1994].
Comment: Also called *Maximum Hypergraph Cut.* Transformation from MAXIMUM NOT-ALL-EQUAL 3-SATISFIABILITY. Variation in which all subsets contain the same number of elements, k, is approximable within 1.138 for $k \leq 3$ and $1/(1-2^{1-k})$ for $k \geq 4$, and not approximable within 1.013 for $k \leq 3$ and $1+O(k^{-3}2^{-k})$ for any $\varepsilon > 0$ for $k \geq 4$ [Kann, Lagergren, and Panconesi, 1996]. It admits a PTAS if $|C| = \Theta(|S|^k)$ [Arora, Karger, and Karpinski, 1995].
Garey and Johnson: SP4

SP4. MINIMUM SET COVER

INSTANCE: Collection C of subsets of a finite set S.
SOLUTION: A set cover for S, i.e., a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C'.
MEASURE: Cardinality of the set cover, i.e., $|C'|$.

Good News: Approximable within $1 + \ln |S|$ [Johnson, 1974].
Bad News: Not in APX [Lund and Yannakakis, 1994] and [Bellare, Goldwasser, Lund, and Russell, 1993].
Comment: Approximable within $\ln(|S|/opt) + O(\ln\ln(|S|/opt)) + O(1)$ [Srinivasan, 1995]. Not approximable within $(1-\varepsilon)\ln|S|$ for any $\varepsilon > 0$, unless NP$\subset$ DTIME$(n^{\log\log n})$ [Feige, 1996]. Equivalent to MINIMUM DOMINATING SET under L-reduction and equivalent to MINIMUM HITTING SET [Ausiello, D'Atri, and Protasi, 1980]. If it is NP-hard to approximate within $\Omega(\log|S|)$, then it is complete for the class of log-approximable problems [Khanna, Motwani, Sudan, and Vazirani, 1994]. Variation in which the subsets have positive weights and the objective is to minimize the sum of the weights in a set cover is also approximable within $1+\ln|S|$ [Chvátal, 1979]. The problem MINIMUM k-SET COVER, the variation in which the cardinality of all sets in C are bounded from above by a constant k is APX-complete and is approximable, for any $\varepsilon > 0$, within $1.4+\varepsilon$ for $k = 3$ and $\sum_{i=1}^{k}\frac{1}{i} - 1/3$ for $k > 3$ [Halldórsson, 1996]. Variation in which the number of occurrences in C of any element is bounded by a constant B is APX-complete [Papadimitriou and Yannakakis, 1991] and approximable within B even for the weighted problem [Hochbaum, 1982]. If, moreover, each element needs to be covered by at least K sets with K constant, this variation is approximable within $B - K + 1$ [Peleg, Schechtman, and Wool, 1993].
It is approximable within 2 if the set system (S, C) is tree representable [Garg, Vazirani, and Yannakakis, 1993]. The constrained variation in which the input is extended with a positive integer k and a subset T of C, and the problem is to find the set cover of size k that contains the largest number of subsets from T, is not approximable within $|C|^{\varepsilon}$ for some $\varepsilon > 0$ [Zuckerman, 1993]. Variation in which a distance matrix between pairs of elements

in S is given and the measure of the solution is not the cardinality of the cover but its diameter (i.e., the maximum distance of any pair of elements in C') is not approximable within any constant in the general case. If the distance matrix satisfies the triangle inequality then this variation can be approximated within 2 and no better approximation is possible. Similar results hold if the cover can be partitioned into clusters and the goal is to minimize the maximum diameter over all clusters [Arkin and Hassin, 1992] (see also MINIMUM k-CLUSTERING).

Garey and Johnson: SP5

SP5. MINIMUM EXACT COVER

INSTANCE: Collection C of subsets of a finite set S.

SOLUTION: A set cover for S, i.e., a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C'.

MEASURE: Sum of cardinalities of the subsets in the set cover, i.e., $\sum_{c \in C'} |c|$.

Good News: Approximable within $1 + \ln |S|$ [Johnson, 1974].

Bad News: As hard to approximate as MINIMUM SET COVER [Lund and Yannakakis, 1994].

Comment: Transformation from MINIMUM SET COVER. The only difference between MINIMUM SET COVER and MINIMUM EXACT COVER is the definition of the objective function.

SP6. MINIMUM TEST COLLECTION

INSTANCE: Collection C of subsets of a finite set S.

SOLUTION: A subcollection $C' \subseteq C$ such that for each pair of distinct elements $x_1, x_2 \in S$ there is some set $c \in C'$ that contains exactly one of x_1 and x_2.

MEASURE: Cardinality of the subcollection, i.e., $|C'|$.

Good News: Approximable within $1 + 2\ln |S|$.

Comment: Transformation to MINIMUM SET COVER [Kann, 1992b]. Observe that every solution has cardinality at least $\lceil \log |S| \rceil$.

Garey and Johnson: SP6

SP7. MINIMUM HITTING SET

INSTANCE: Collection C of subsets of a finite set S.

SOLUTION: A hitting set for C, i.e., a subset $S' \subseteq S$ such that S' contains at least one element from each subset in C.

MEASURE: Cardinality of the hitting set, i.e., $|S'|$.

Good News: See MINIMUM SET COVER.

Bad News: See MINIMUM SET COVER.

Comment: Equivalent to MINIMUM SET COVER [Ausiello, D'Atri, and Protasi, 1980]. Therefore approximation algorithms and nonapproximability results for MINIMUM SET COVER will carry over to MINIMUM HITTING SET. The constrained variation in which the input is extended with a subset T of S, and the problem is to find the hitting set that contains the largest number of elements from T, is not approximable within $|S|^{\varepsilon}$ for some $\varepsilon > 0$

[Zuckerman, 1993]. Several special cases in which, given compact subsets of R^d, the goal is to find a set of straight lines of minimum cardinality so that each of the given subsets is hit by at least one line, are approximable [Hassin and Megiddo, 1991].

Garey and Johnson: SP8

SP8. MINIMUM GEOMETRIC DISK COVER

INSTANCE: Set $P \subseteq Z \times Z$ of points in the plane, a rational number $r > 0$.

SOLUTION: A set of centers $C \subseteq Q \times Q$ in the Euclidean plane, such that every point in P will covered by a disk with radius r and center in one of the points in C.

MEASURE: Cardinality of the disk cover, i.e., $|C|$.

Good News: Admits a PTAS [Hochbaum and Maass, 1985].

Comment: There is a PTAS also for other convex objects, e.g. squares, and in d-dimensional Euclidean space for $d \geq 3$ [Hochbaum and Maass, 1985]. Variation in which the objective is to cover a set of points on the line by rings of given inner radius r and width w, admits a PTAS with time complexity exponential in r/w, for arbitrary r and w the problem is approximable with relative error at most $1/2$ [Hochbaum and Maass, 1987].

The *Maximum Geometric Square Packing Problem*, where the objective is to place as many squares of a given size within a region in the plane, also admits a PTAS [Hochbaum and Maass, 1985].

Weighted Set Problems

SP9. MAXIMUM CONSTRAINED PARTITION

INSTANCE: Finite set A and a size $s(a) \in Z^+$ for each $a \in A$, element $a_0 \in A$, and a subset $S \subseteq A$.

SOLUTION: A partition of A, i.e., a subset $A' \subseteq A$ such that $\sum_{a \in A'} s(a) = \sum_{a \in A-A'} s(a)$.

MEASURE: Number of elements from S on the same side of the partition as a_0.

Bad News: Not approximable within $|A|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].

Garey and Johnson: Similar to SP12

SP10. MINIMUM 3-DIMENSIONAL ASSIGNMENT

INSTANCE: Three sets X, Y, and W and a cost function $c : X \times Y \times W \to N$.

SOLUTION: An assignment A, i.e., a subset $A \subseteq X \times Y \times W$ such that every element of $X \cup Y \cup W$ belongs to exactly one triple in A.

MEASURE: The cost of the assignment, i.e., $\sum_{(x,y,w) \in A} c(x, y, w)$.

Bad News: Not in APX [Crama and Spieksma, 1992].

Comment: The negative result holds even if c is either defined as $c(x, y, w) = d(x, y) + d(x, w) + d(y, w)$ or defined as $c(x, y, w) = \min\{d(x, y) + d(x, w), d(x, y) + d(y, w), d(x, w) + d(y, w)\}$ where d is any distance function. In these cases, however, the problem is approximable within $4/3$ if d satisfies the triangle inequality. Similar results hold for the k-dimensional problem [Bandelt, Crama, and Spieksma, 1991].

Garey and Johnson: Weighted version of SP2

SP11. MAXIMUM CAPACITY REPRESENTATIVES

INSTANCE: Disjoint sets $S_1, \ldots, S_m$ and, for any $i \neq j$, $x \in S_i$, and $y \in S_j$, a nonnegative capacity $c(x, y)$.
SOLUTION: A system of representatives T, i.e., a set T such that, for any i, $|T \cap S_i| = 1$.
MEASURE: The capacity of the system of representatives, i.e., $\sum_{x,y \in T} c(x, y)$.

Bad News: Not approximable within $2^{\theta(\log^p n)}$ for some $p > 0$ unless NP$\subseteq$ DTIME$(n^{d \log^{1/\varepsilon} n})$ [Bellare, 1993].
Comment: Does not admit a PTAS.

Storage and Retrieval

Data Storage

SR1. MINIMUM BIN PACKING

INSTANCE: Finite set U of items, a size $s(u) \in Z^+$ for each $u \in U$, and a positive integer bin capacity B.
SOLUTION: A partition of U into disjoint sets $U_1, U_2, \ldots, U_m$ such that the sum of the a partition of U such that the sum of the items in each U_i is B or less.
MEASURE: The number of used bins, i.e., the number of disjoint sets, m.

Good News: Approximable within 3/2 [Simchi-Levi, 1994] and within $71/60+\frac{78}{71 opt}$ [Johnson and Garey, 1985], [Yue, 1991].
Bad News: Not approximable within $3/2-\varepsilon$ for any $\varepsilon > 0$ [Garey and Johnson, 1979].
Comment: Admits an FPTAS$^\infty$, that is, is approximable within $1 + \varepsilon$ in time polynomial in $1/\varepsilon$, where $\varepsilon = O(\log^2(opt)/opt)$ [Karmarkar and Karp, 1982]. APX-intermediate unless the polynomial-hierarchy collapses [Crescenzi, Kann, Silvestri, and Trevisan]. A survey of approximation algorithms for MINIMUM BIN PACKING is found in [Coffman, Garey, and Johnson, 1984]. If a partial order on U is defined and we require the bin packing to obey this order, then the problem is approximable within 2 [Wee and Magazine, 1982], and is not in FPTAS$^\infty$ [Queyranne, 1985]. The generalization in which the cost of a bin is a monotone and concave function of the number of items in the bin is approximable within 7/4 and is not approximable within 4/3 unless some information about the cost function is used [Anily, Bramel, and Simchi-Levi, 1994]. The generalization of this problem in which a conflict graph is given such that adjacent items are assigned to different bins is approximable within 2.7 for graphs that can be colored in polynomial time [Jansen and Öhring, 1997] and not approximable within $|U|^\varepsilon$ for a given ε in the general case [Lund and Yannakakis, 1994].
Garey and Johnson: SR1

SR2. MINIMUM HEIGHT TWO DIMENSIONAL PACKING

INSTANCE: Set of rectangles $B = \{(x_i, y_i)\}$ with positive sizes (width $x_i \leq 1$ and height y_i).
SOLUTION: A packing P of the rectangles in B into a unit-width bin with infinite height. The rectangles must be packed orthogonally and may not be rotated.
MEASURE: Height of the packing P.

Good News: Approximable within 2 [Schiermeyer, 1994].
Comment: Also called *Minimum Strip Packing.* Approximable within $1+\varepsilon+O(\max y_i/\varepsilon^2)/opt$ in time polynomial in $1/\varepsilon$ for any $\varepsilon > 0$ [Kenyon and Rémila, 1996]. Variation in which the dimensions of the rectangles are bounded below by a constant admits a PTAS [Fernandez de la Vega and Zissimopoulos, 1991]. The three-dimensional variation is approximable within 3.25 [Li and Cheng, 1990].

SR3. MINIMUM DYNAMIC STORAGE ALLOCATION

INSTANCE: Set A of items to be stored, each $a \in A$ having a size $s(a) \in Z^+$, an arrival time $r(a) \in Z^+$, and a departure time $d(a) \in Z^+$.
SOLUTION: A feasible allocation of storage for A, i.e., a function $\sigma : A \to Z^+$ such that, for all $a, a' \in A$, if $I(a) \cap I(a')$ is nonempty then either $d(a) \leq r(a')$ or $d(a') \leq r(a)$.
MEASURE: The size of the allocation, i.e., $\max_{a\in A}(\sigma(a) + s(a) - 1)$.

Good News: Approximable within 6 [Slusarek, 1989 and Kierstead, 1991].
Comment: Approximable within 3 if the sizes are powers of 2 [Slusarek, 1989].

Compression and Representation

SR4. SHORTEST COMMON SUPERSEQUENCE

INSTANCE: Finite alphabet Σ, finite set R of strings from Σ^*.
SOLUTION: A string $w \in \Sigma^*$ such that each string $x \in R$ is a subsequence of w, i.e. one can get x by taking away letters from w.
MEASURE: Length of the supersequence, i.e., $|w|$.

Bad News: Not in APX [Jiang and Li, 1994b].
Comment: Transformation from MINIMUM FEEDBACK VERTEX SET and self-improvability. Not approximable within $\log^\delta |R|$ for some $\delta > 0$ unless NP$\subset$ DTIME($n^{\text{polylog} n}$) [Jiang and Li, 1994b]. APX-complete if the size of the alphabet Σ is fixed [Jiang and Li, 1994b] and [Bonizzoni, Duella, and Mauri, 1994]. Variation in which the objective is to find the longest minimal common supersequence (a supersequence that cannot be reduced to a shorter common supersequence by removing a letter) is APX-hard even over the binary alphabet, and the *shortest maximal common non-supersequence problem* is APX-hard even over the binary alphabet [Middendorf, 1994]. Variation in which the longest common subsequence is known is also APX-hard even over the binary alphabet [dAnzeo, 1996].
Garey and Johnson: SR8

SR5. SHORTEST COMMON SUPERSTRING

INSTANCE: Finite alphabet Σ, finite set R of strings from Σ^*.
SOLUTION: A string $w \in \Sigma^*$ such that each string $x \in R$ is a substring of w, i.e. $w = w_0 x w_1$ where $w_0, w_1 \in \Sigma^*$.
MEASURE: Length of the superstring, i.e., $|w|$.

Good News: Approximable within 2.75 [Armen and Stein, 1994].

Bad News: APX-complete [Blum, Jiang, Li, Tromp, and Yannakakis, 1994].

Comment: Transformation from MINIMUM METRIC TRAVELING SALESPERSON PROBLEM with distances one and two. Variation in which there are negative strings in the input and a solution cannot contain any negative string as a substring, is approximable within $O(\log opt)$ [Li, 1990]. If the number of negative strings is constant, or if no negative strings contain positive strings as substrings, the problem is approximable within a constant [Jiang and Li, 1994a]. The complementary MAXIMUM COMPRESSION problem, where the objective is to maximize $\sum_{x \in R} |x| - |w|$, is approximable within 2 [Tarhio and Ukkonen, 1988] and [Turner, 1989].

Garey and Johnson: SR9

SR6. LONGEST COMMON SUBSEQUENCE

INSTANCE: Finite alphabet Σ, finite set R of strings from Σ^*.

SOLUTION: A string $w \in \Sigma^*$ such that w is a subsequence of each $x \in R$, i.e. one can get w by taking away letters from each x.

MEASURE: Length of the subsequence, i.e., $|w|$.

Good News: Approximable within $O(m/\log m)$, where m is the length of the shortest string in R [Halldórsson, 1995b].

Bad News: Not approximable within $n^{1/4-\varepsilon}$ for any $\varepsilon > 0$, where in is the maximum of $|R|$ and $|\Sigma|$ [Berman and Schnitger, 1992], [Jiang and Li, 1994b] and [Bellare, Goldreich, and Sudan, 1995].

Comment: Transformation from MAXIMUM INDEPENDENT SET. APX-complete if the size of the alphabet Σ is fixed [Jiang and Li, 1994b] and [Bonizzoni, Duella, and Mauri, 1994]. Variation in which the objective is to find the shortest maximal common subsequence (a subsequence that cannot be extended to a longer common subsequence) is APX-hard even over the binary alphabet [Middendorf, 1994].

Garey and Johnson: SR10

SR7. MAXIMUM COMMON SUBTREE

INSTANCE: Collection $T_1, \ldots, T_n$ of trees.

SOLUTION: A tree T' isomorphic to a subtree (i.e. induced connected subgraph) of each T_i.

MEASURE: Size, i.e. number of nodes, of the common subtree T'.

Good News: Approximable within $O(n(\log\log n/\log n)^2)$, where n is the size of the smallest T_i [Khanna, Motwani, and Yao, 1995].

Bad News: Not approximable within n^ε for some $\varepsilon > 0$ [Akutsu and Halldórsson, 1994].

Comment: Transformation from MAXIMUM INDEPENDENT SET. Hardness holds also for the variations where the nodes are labeled, where maximum degree is bounded by a constant $B \geq 3$, and where the tree is rooted and the solution must contain the root [Akutsu and Halldórsson, 1994]. Approximable within $O(n \log\log n/\log^2 n)$ when the maximum degree is constant [Khanna, Motwani, and Yao, 1995]. Variation in which the nodes are labeled is approximable within $O(n(\log\log n/\log n)^2)$ if the number of distinct labels is $O(log^{O(1)} n)$ [Khanna, Motwani, and Yao, 1995] and within $O(n/\log n)$ in general [Akutsu and Halldórsson, 1994]. Approximable within 2 if the solution is an isomorphic edge subset of each tree. APX-hard when the number of input trees is a constant at least 3, but polynomial-time solvable if the maximum degree is additionally bounded by constant or for only two trees [Akutsu and Halldórsson, 1994].

SR8. MAXIMUM COMMON POINT SET

INSTANCE: Positive integer d, collection $S_1, \dots, S_k$ of d-dimensional point sets.
SOLUTION: A point set S' congruent to a subset of each set S_i in the collection.
MEASURE: Size, i.e. number of elements, of the common point set S'.

Good News: Approximable within $O(n/\log n)$, where n is the size of the smallest S_i [Akutsu and Halldórsson, 1994].
Bad News: Not approximable within n^ε for some $\varepsilon > 0$ [Akutsu and Halldórsson, 1994].
Comment: Transformation from MAXIMUM INDEPENDENT SET. The bad news is valid even if a small gap is allowed between corresponding points.

SR9. MINIMUM RECTANGLE COVER

INSTANCE: An $n \times n$ binary matrix.
SOLUTION: A collection of m rectangles that covers precisely those entries in M that are 1's.
MEASURE: Size, i.e. number m of elements, of the collection.

Good News: Approximable within 14 [Levcopoulos and Gudmundsson, 1996].
Garey and Johnson: SR25

Miscellaneous

SR10. MINIMUM TREE COMPACT PACKING

INSTANCE: A tree $T = \langle V, E\rangle$, a node-weight function $w : V \to Q^+$ such that $\sum_{v\in V} w(v) = 1$, and a page-capacity p.
SOLUTION: A compact packing of T into pages of capacity p, i.e., a function $\tau : V \to Z^+$ such that $|\tau^{-1}(i)| = p$.
MEASURE: The number of page faults of the packing, i.e., $\sum_{v\in V} c_\tau(v)w(v)$ where

$$c_\tau(v) = \sum_{i=0}^{l(v)-1} \Delta_\tau(v_i),$$

$l(v)$ denotes the number of edges in the path from the root to v, v_i denotes the ith node in this path, and $\Delta_\tau(v)$ is equal to 0 if the parent of v is assigned the same page of u, it is equal to 1 otherwise.

Good News: Approximable within absolute error 1 [Gil and Itai, 1995].
Comment: If all $w(v)$ are equal then it is approximable within absolute error 1/2.

Sequencing and Scheduling

Sequencing on One Processor

SS1. MAXIMUM CONSTRAINED SEQUENCING TO MINIMIZE TARDY TASK WEIGHT

INSTANCE: Set T of tasks, for each task $t \in T$ a length $l(t) \in Z^+$, a weight $w(t) \in Z^+$, and a deadline $d(t) \in Z^+$, a subset $S \subseteq T$, and a positive integer K.
SOLUTION: A one-processor schedule σ for T such that the sum of $w(t)$, taken over all $t \in T$ for which $\sigma(t) + l(t) > d(t)$ does not exceed K.
MEASURE: Cardinality of jobs in S completed by the deadline.

Bad News: Not approximable within $|T|^\varepsilon$ for some $\varepsilon > 0$ [Zuckerman, 1993].
Garey and Johnson: Similar to SS3

SS2. MINIMUM STORAGE-TIME SEQUENCING

INSTANCE: Set T of tasks, a directed acyclic graph $G = \langle T, E\rangle$ defining precedence constraints for the tasks, for each task a length $l(t) \in Z^+$, and for each edge in the graph a weight $w(t_1, t_2)$ measuring the storage required to save the intermediate results generated by task t_1 until it is consumed by task t_2.
SOLUTION: A one-processor schedule for T that obeys the precedence constraints, i.e., a permutation $\pi : [1..|T|] \to [1..|T|]$ such that, for each edge $\langle t_i, t_j\rangle \in E$, $\pi^{-1}(i) < \pi^{-1}(j)$.
MEASURE: The total storage-time product, i.e.,

$$\sum_{\langle t_{\pi(i)}, t_{\pi(j)}\rangle \in E} w(t_{\pi(i)}, t_{\pi(j)}) \sum_{k=\min(i,j)}^{\max(i,j)} l(t_{\pi(k)}).$$

Good News: Approximable within $O\left(\log |T| \log \sum_{t\in T} l(t)\right)$ [Ravi, Agrawal, and Klein, 1991].

SS3. MINIMUM PRECEDENCE CONSTRAINED SEQUENCING WITH DELAYS

INSTANCE: Set T of tasks, a directed acyclic graph $G = \langle T, E\rangle$ defining precedence constraints for the tasks, a positive integer D, and for each task an integer delay $0 \le d(t) \le D$.
SOLUTION: A one-processor schedule for T that obeys the precedence constraints and the delays, i.e., an injective function $S : T \to Z^+$ such that, for each edge $\langle t_i, t_j\rangle \in E$, $S(t_j) - S(t_i) > d(t_i)$.
MEASURE: The finish time for the schedule, i.e., $\max_{t\in T} S(t)$.

Good News: Approximable within $2 - 1/(D + 1)$ [Bernstein, Rodeh, and Gertner, 1989].
Comment: Approximable within $2 - 2/(D + 1)$ if $d(t) \in \{0, D\}$ for all tasks t.

SS4. MINIMUM SEQUENCING WITH RELEASE TIMES

INSTANCE: Set T of tasks, for each task $t \in T$ a release time $r(t) \in Z^+$, a length $l(t) \in Z^+$, and a weight $w(t) \in Z^+$.
SOLUTION: A one-processor schedule for T that obeys the release times, i.e. a function $f : T \to N$ such that, for all $u \ge 0$, if $S(u)$ is the set of tasks t for which $f(t) \le u < f(t) + l(t)$, then $|S(u)| = 1$ and for each task t, $f(t) \ge r(t)$.
MEASURE: The weighted sum of completion times, i.e. $\sum_{t\in T} w(t)(f(t) + l(t))$.

Good News: Approximable within 2 [Goemans, 1997].
Comment: Approximable within $e/(e-1) \approx 1.58$ if all weights $w(t) = 1$ [Chekuri, Motwani, Natarajan, and Stein, 1997]. Variation in which there are precedence constraints on T instead of release times is approximable within 2 [Hall, Schulz, Shmoys, and Wein, 1997].

SS5. MINIMUM TIME-COST TRADEOFF

INSTANCE: Set J of activities, a directed acyclic graph defining precedence constraints for the activities, a positive budget B, and for each activity $j \in J$ a non-increasing cost function c_j described as a step function with l_j steps:

$$c_j(t) = \begin{cases} \infty & \text{if } 0 \le t < a_{j,1}, \\ c_j(a_{j,i}) & \text{if } a_{j,i} \le t < a_{j,i+1}, 1 \le i < l_j, \\ 0 & \text{if } a_{j,l_j} \le t, \end{cases}$$

where $\infty > c_j(a_{j,1}) > \cdots > c_j(a_{j,l_j})$.
SOLUTION: A one-processor schedule for J that obeys the precedence constraints and activities durations $t_j > 0$ for each $j \in J$ that obeys the budget, i.e. $\sum_{j \in J} c_j(t_j) \le B$.
MEASURE: The total duration of all activities, i.e., $\sum_{j \in J} t_j$.

Good News: Approximable within $(3/2)\log f + (7/2)$, where f is the ratio of the maximum allowed duration of any activity to the minimum allowed non-zero duration of any activity [Skutella, 1997].
Comment: Variation in which $a_{j,i} \in \{0, 1, 2\}$ for all i, j is approximable within 3/2 but not approximable within $3/2 - \varepsilon$ for any $\varepsilon > 0$ [Skutella, 1997].

Multiprocessor Scheduling

SS6. MINIMUM MULTIPROCESSOR SCHEDULING

INSTANCE: Set T of tasks, number m of processors, length $l(t,i) \in Z^+$ for each task $t \in T$ and processor $i \in [1..m]$.
SOLUTION: An m-processor schedule for T, i.e., a function $f : T \to [1..m]$.
MEASURE: The finish time for the schedule, i.e., $\max_{i \in [1..m]} \sum_{\substack{t \in T: \\ f(t)=i}} l(t,i)$.

Good News: Approximable within 2 [Lenstra, Shmoys, and Tardos, 1990].
Bad News: Not approximable within $3/2-\varepsilon$ for any $\varepsilon > 0$ [Lenstra, Shmoys, and Tardos, 1990].
Comment: Admits an FPTAS for the variation in which the number of processors m is constant [Horowitz and Sahni, 1976]. Admits a PTAS for the uniform variation, in which $l(t,i)$ is independent of the processor i [Hochbaum and Shmoys, 1987]. A variation in which, for each task t and processor i, a cost $c(t,i)$ is given in input and the goal is to minimize a weighted sum of the finish time and the cost is approximable within 2 [Shmoys and Tardos, 1993].
Garey and Johnson: SS8

SS7. MINIMUM PRECEDENCE CONSTRAINED SCHEDULING

INSTANCE: Set T of tasks, each having length $l(t) = 1$, number m of processors, and a partial order $<$ on T.

SOLUTION: An m-processor schedule for T that obeys the precedence constraints, i.e., a function $f : T \to N$ such that, for all $u \geq 0$, $|f^{-1}(u)| \leq m$ and such that $t < t'$ implies $f(t') > f(t)$.

MEASURE: The finish time for the schedule, i.e., $\max_{t \in T} f(t)$.

Good News: Approximable within $2 - 2/|T|$ [Lam and Sethi, 1977].

Bad News: Not approximable within $4/3-\varepsilon$ for any $\varepsilon > 0$ [Lenstra and Rinnooy Kan, 1978].

Comment: A variation with an enlarged class of allowable constraints is approximable within $3 - 4/(|T| + 1)$ while a variation in which the partial order $<$ is substituted with a weak partial order $\leq$ is approximable within $2 - 2/(|T| + 1)$ [Berger and Cowen, 1991]. Variation in which there is a communication delay of 1, i.e., if two tasks $t < t'$ are scheduled on different machines, then $f(t') > f(t) + 1$, is approximable within 3 [Rayward-Smith, 1987], and not approximable within $5/4-\varepsilon$ [Hoogeveen, Lenstra, and Veltman, 1995]. The same problem without limit on the number of processors, i.e. with $m = \infty$, is not approximable within $7/6-\varepsilon$ [Hoogeveen, Lenstra, and Veltman, 1995]. Generalization where the tasks have lengths $l(t)$ and the processors have speed factors $s(i)$ is approximable within $O(\log m)$ [Chudak and Shmoys, 1997].

Garey and Johnson: SS9

SS8. MINIMUM RESOURCE CONSTRAINED SCHEDULING

INSTANCE: Set T of tasks each having length $l(t)$, number m of processors, number r of resources, resource bounds b_i, $1 \leq i \leq r$, and resource requirement $r_i(t)$, $0 \leq r_i(t) \leq b_i$, for each task t and resource i.

SOLUTION: An m processor schedule for T that obeys the resource constraints, i.e., a function $f : T \to Z$ such that for all $u \leq 0$, if $S(u)$ is the set of tasks t for which $f(t) \leq u < f(t)+l(t)$, then $|S(u)| \leq m$ and for each resource i

$$\sum_{t \in S(u)} r_i(t) \leq b_i.$$

MEASURE: The makespan of the schedule, i.e., $\max_{t \in T}(f(t) + l(t))$.

Good News: Approximable within 2 [Garey and Graham, 1975].

Comment: Note that the restriction in which there is only one resource, i.e., the available processors, is identical to minimizing the makespan of the schedule of parallel tasks on m processors. In this case, minimizing the average response time, i.e., $\frac{1}{|T|} \sum_{t \in T}(f(t) + l(t))$ is approximable within 32 [Turek, Schwiegelshohn, Wolf, and Yu, 1994]. The further variation in which each task can be executed by any number of processors and the length of a task is a function of the number of processors allotted to it is also approximable [Ludwig and Tiwari, 1994]. Variation in which every task is of length 1 and has an integer ready-time, which means that it cannot be processed before its ready-time, is approximable [Srivastav and Stangier, 1994].

Garey and Johnson: SS10

SS9. Minimum Preemptive Scheduling with Set-Up Times

Instance: Set C of compilers, set T of tasks, number m of processors, length $l(t) \in Z^+$ and compiler $c(t) \in C$ for each $t \in T$, set-up time $s(c) \in Z^+$ for each $c \in C$.

Solution: An m-processor preemptive schedule for T, i.e., for each $t \in T$, a partition of t into any number of subtasks $t_1, \ldots, t_k$ such that $\sum_{i=1}^{k} l(t_i) = l(t)$ and an assignment $\sigma = (\sigma_1, \sigma_2)$ that assigns to each subtask t_i of t a pair of nonnegative integers $(\sigma_1(t_i), \sigma_2(t_i))$ such that $1 \le \sigma_1(t_i) \le m$ and, for $1 \le i < k$, $\sigma_2(t_{i+1}) \ge \sigma_2(t_i) + l(t_i)$. Such a schedule must satisfy the following additional constraint: Whenever two subtasks t_i of t and t'_j of t' with $\sigma_2(t_i) < \sigma_2(t'_j)$ are scheduled consecutively on the same machine (i.e., $\sigma_1(t_i) = \sigma_1(t'_j)$ and no other subtask t''_k has $\sigma_1(t''_k) = \sigma_1(t_i)$ and $\sigma_2(t_i) < \sigma_2(t''_k) < \sigma_2(t'_j)$), then $\sigma_2(t'_j) \ge \sigma_2(t_i) + l(t_i)$ if they have the same compiler (i.e., $c(t) = c(t')$) and $\sigma_2(t'_j) \ge \sigma_2(t_i) + l(t_i) + s(c(t'))$ if they have different compilers.

Measure: The overall completion time, i.e., the maximum over all subtasks of $\sigma_2(t_i) + l(t_i)$.

Good News: Approximable within

$$\max\left(\frac{3m}{2m+1}, \frac{3m-4}{2m-2}\right)$$

[Chen, 1993].

Comment: If all set-up times are equal, then the problem is approximable within $3/2 - 1/2m$ for $m \ge 2$ and admits an Fptas for $m = 2$ [Wöginger and Yu, 1992].

Garey and Johnson: SS6 and SS12

SS10. Minimum Multiprocessor Scheduling with Speed Factors

Instance: Set T of tasks, number m of processors, for each task $t \in T$ a length $l(t) \in Z^+$, and for each processor $i \in [1..m]$ a speed factor $s(i) \in Q$ such that $s(1) = 1$ and $s(i) \ge 1$ for every i.

Solution: An m-processor schedule for T, i.e., a function $f : T \to [1..m]$.

Measure: The finish time for the schedule, i.e., $\max_{i \in [1..m]} \sum_{\substack{t \in T: \\ f(t)=i}} l(t)/s(i)$.

Good News: Admits a Ptas [Hochbaum and Shmoys, 1988].

Bad News: Does not admit an Fptas [Hochbaum and Shmoys, 1988].

Comment: Admits an Fptas for the variation in which the number of processors m is constant [Horowitz and Sahni, 1976].

SS11. Minimum Parallel Processor Total Flow Time

Instance: Set T of tasks, number m of identical processors, for each task $t \in T$ a release time $r(t) \in Z^+$ and a length $l(t) \in Z^+$.

Solution: An m-processor schedule for T that obeys the resource constraints and the release times, i.e., a function $f : T \to N \times [1..m]$ such that, for all $u \ge 0$ and for each processor i, if $S(u, i)$ is the set of tasks t for which $f(t)_1 \le u < f(t)_1 + l(t)$ and $f(t)_2 = i$, then $|S(u, i)| = 1$ and for each task t, $f(t)_1 \ge r(t)$.

Measure: The total flow time for the schedule, i.e., $\sum_{t \in T} (f(t)_1 - r(t))$.

Good News: Approximable within $O\left(\sqrt{n/m}\log(n/m)\right)$ where $n = |T|$ [Leonardi and Raz, 1997].

Bad News: Not approximable within $O(n^{1/3-\varepsilon})$ for any $\varepsilon > 0$ [Leonardi and Raz, 1997].

Comment: In the case $m = 1$, it is approximable within $\sqrt{n}$ and it is not approximable within $O(n^{1/2-\varepsilon}$ for any $\varepsilon > 0$ [Kellerer, Tautenhahn, and Woeginger, 1996]. In the preemptive case, that is, in the case a job that is running can be preempted and continue later on any machine, the problem is approximable within $O(\log(\min\{n/m, P\}))$ and it is not approximable within $O(\log(\max\{n/m, P\}))$ where $P = \max_{s,t\in T} l(s)/l(t)$ [Leonardi and Raz, 1997]. Variation in which all speed factors are 1 and the load is measured using the L_p norm, i.e. $\max_{[1..m]}(\sum l(t)^p)^{1/p}$, admits a PTAS for any $p \geq 1$ [Alon, Azar, Woeginger, and Yadid, 1997].

SS12. MINIMUM WEIGHTED COMPLETION TIME SCHEDULING

INSTANCE: Set T of tasks, number m of identical processors, for each task $t \in T$ a release time $r(t) \in Z^+$ and a length $l(t) \in Z^+$.

SOLUTION: An m-processor schedule for T that obeys the resource constraints and the release times, i.e., a function $f : T \to N \times [1..m]$ such that, for all $u \geq 0$ and for each processor i, if $S(u, i)$ is the set of tasks t for which $f(t)_1 \leq u < f(t)_1 + l(t)$ and $f(t)_2 = i$, then $|S(u, i)| = 1$ and for each task t, $f(t)_1 \geq r(t)$.

MEASURE: The weighted sum of completion times, i.e. $\sum_{t\in T} w(t)(f(t)_1 + l(t))$.

Good News: Approximable within 2.85 [Chekuri, Motwani, Natarajan, and Stein, 1997].

Comment: The preemptive case is approximable within 2 [Phillips, Stein, and Wein, 1995]. Generalization (of the nonpreemptive case) where there are precedence constraints on T that must be obeyed in the solution is approximable within 5.33 [Chakrabati, Phillips, Schulz, Shmoys, Stein, and Wein, 1996].

Garey and Johnson: SS13

SS13. MINIMUM 3-DEDICATED PROCESSOR SCHEDULING

INSTANCE: Set T of tasks, set P of 3 processors, and, for each task $t \in T$, a length $l(t) \in Z^+$ and a required subset of processors $r(t) \subseteq P$.

SOLUTION: A schedule for T, i.e., a starting time function $s : T \to Z^+$ such that, for any two tasks t_1 and t_2 with $r(t_1) \cap r(t_2) \neq \emptyset$, either $s(t_1) + l(t_1) < s(t_2)$ or $s(t_2) + l(t_2) < s(t_1)$.

MEASURE: The makespan of the schedule, i.e., $\max_{t\in T}(s(t) + r(t))$.

Good News: Approximable within 7/6 [Goemans, 1994].

Shop Scheduling

SS14. MINIMUM OPEN-SHOP SCHEDULING

INSTANCE: Number $m \in Z^+$ of processors, set J of jobs, each $j \in J$ consisting of m operations $o_{i,j}$ with $1 \leq i \leq m$ (with $o_{i,j}$ to be executed by processor i), and for each operation a length $l_{i,j} \in N$.

Solution: An open-shop schedule for J, i.e., a collection of one-processor schedules $f_i : J \to N$, $1 \leq i \leq m$, such that $f_i(j) > f_i(j')$ implies $f_i(j) \geq f_i(j') + l_{i,j'}$, such that for each $j \in J$ the intervals $[f_i(j), f_i(j) + l_{i,j})$ are all disjoint.
Measure: The completion time of the schedule, i.e., $\max\limits_{1 \leq i \leq m, j \in J} f_i(j) + l_{i,j}$.

Good News: Approximable within 2 [Lenstra and Shmoys, 1995].
Bad News: Not approximable within $5/4 - \varepsilon$ for any $\varepsilon > 0$ [Williamson et al., 1994].
Comment: Approximable within 3/2 if $m = 3$ [Chen and Strusevich, 1993a]. Variation in which $m = 2$, but the two processors are replaced by respectively m_1 and m_2 identical parallel processors, is approximable within 3/2 if $\min(m_1, m_2) = 1$, 5/3 if $m_1 = m_2 = 2$ and $2 - 2/\max(m_1 + m_2)^2$ otherwise [Chen and Strusevich, 1993b].
Garey and Johnson: SS14

SS15. Minimum Flow-Shop Scheduling

Instance: Number $m \in Z^+$ of processors, set J of jobs, each $j \in J$ consisting of m operations $o_{i,j}$ with $1 \leq i \leq m$ (with $o_{i,j}$ to be executed by processor i), and for each operation a length $l_{i,j} \in N$.
Solution: An open-shop schedule for J (see Minimum Open-Shop Scheduling) such that, for each $j \in J$ and $1 \leq i < m$, $f_{i+1}(j) \geq f_i(j) + l_{i,j}$.
Measure: The completion time of the schedule, i.e., $\max\limits_{j \in J} f_m(j) + l_{m,j}$.

Good News: Approximable within $m/2$ if m is even and within $m/2 + 1/6$ if m is odd [Chen et al., 1995].
Bad News: Not approximable within $5/4 - \varepsilon$ for any $\varepsilon > 0$ [Williamson et al., 1994].
Comment: Approximable within 5/3 if $m = 3$ [Chen et al., 1995]. Variation in which $m = 2$, but the two processors are replaced by respectively m_1 and m_2 identical parallel processors, is approximable within $2 - 1/\max(m_1 + m_2)$ [Chen, 1994].
Garey and Johnson: SS15

SS16. Minimum Two-Processor Flow-Shop Scheduling with Batch Set-Up Times

Instance: Set C of compilers, set J of jobs, for each $j \in J$, which consists of two operations $o_{i,j}$, $i = 1, 2$, a compiler $k(j) \in C$, for each operation $o_{i,j}$ a length $l_{i,j} \in N$ and for each $c \in C$ a pair of set-up times $(s_1(c), s_2(c))$, where $s_i(c) \in Z^+$.
Solution: A two-processor flow-shop schedule for J (see Minimum Flow-Shop Scheduling) such that, whenever two operations $o_{i,j}$ and $o_{i,j'}$ with $f_i(j) < f_i(j')$ are scheduled consecutively (i.e., no other operation $o_{i,j''}$ has $f_i(j) < f_i(j'') < f_i(j')$) and have different compilers (i.e., $k(j) \neq k(j')$), then $f_i(j') \geq f_i(j) + l_{i,j} + s_i(k(j'))$.
Measure: The completion time of the schedule, i.e., $\max\limits_{j \in J} f_2(j) + l_{2,j}$.

Good News: Approximable within 4/3 [Chen, Potts, and Strusevich, 1995].

SS17. Minimum Job Shop Scheduling

Instance: Number $m \in Z^+$ of processors, set J of jobs, each $j \in J$ consisting of a sequence of n_j operations $o_{i,j}$ with $1 \leq i \leq n_j$, for each such operation a processor $p_{i,j} \in [1..m]$ and a length $l_{i,j} \in N$.

Solution: A job shop schedule for J, i.e., a collection of one-processor schedules $f_p : \{o_{i,j} : p_{i,j} = p\} \to N$ such that $f_p(o_{i,j}) > f_p(o_{i',j'})$ implies $f_p(o_{i,j}) \geq f_p(o_{i',j'}) + l_{i',j'}$ and such that $f_p(o_{i+1,j}) \geq f_p(o_{i,j}) + l_{i,j}$.
Measure: The completion time of the schedule, i.e., $\max_{j \in J} f_p(o_{n_j,j}) + l_{n_j,j}$.

Good News: Approximable within $O\left(\lceil \log(\min(m\mu, l_{\max}))/\log\log(m\mu)\rceil \log(m\mu)/\log\log(m\mu)\right)$, where $\mu = \max_{j\in J} n_j$ and $l_{\max} = \max l_{i,j}$ [Goldberg, Paterson, Srinivasan, and Sweedyk, 1997].
Bad News: Not approximable within $5/4-\varepsilon$ for any $\varepsilon > 0$ [Williamson et al., 1994].
Comment: Transformation from 3-Partition. Approximable within $O(\log^2 m)$ if each job must be processed on each machine at most once [Shmoys, Stein, and Wein, 1994]. The variation in which the operations must be processed in an order consistent to a particular partial order, and the variation in which there are different types of machines, for each type, there are a specified number of identical processors, and each operation may be processed on any processor of the appropriate type, are approximable within $O(\log^2(m\mu))$ [Shmoys, Stein, and Wein, 1994].
Garey and Johnson: SS18

Miscellaneous

SS18. Minimum File Transfer Scheduling

Instance: A file transfer graph, i.e., a graph $G = \langle V, E\rangle$, a port constraint function $p : V \to N$ and a file length function $L : E \to N$.
Solution: A file transfer schedule, i.e., a function $s : E \to N$ such that, for each vertex v and for each $t \in N$,

$$|\{u : (u,v) \in E \wedge s(e) \leq t \leq s(e) + L(e)\}| \leq p(v).$$

Measure: The makespan of the schedule, i.e., $\max_{e\in E}(s(e) + L(e))$.

Good News: Approximable within 2.5 [Coffman, Garey, Johnson, and Lapaugh, 1985].
Comment: Several special cases with better guarantees are also obtainable [Coffman, Garey, Johnson, and Lapaugh, 1985].

SS19. Minimum Schedule Length

Instance: A network $N = \langle V, E, b, c\rangle$ where $G = \langle V, E\rangle$ is a graph, $b : V \to N$ is the vertex-capacity function, and $c : E \to N$ is the edge-capacity function, and a set T of tokens $t = \langle u, v, p\rangle$ where $u, v \in V$ and p is either a path from u to v or the empty set.
Solution: A schedule S, i.e., a sequence $f_0, \ldots, f_l$ of configuration functions $f_i : T \to V$ such that

1. for any token $t = \langle u, v, p\rangle$, $f_0(t) = u$ and $f_l(t) = v$,
2. for any $0 \leq i \leq l-1$ and for any token t, if $f_i(t) = v$ and $f_{i+1}(t) = w$ then (a) $(u,v) \in E$, (b) $|\{t' : f_i(t') = w\}| < b(w)$, (c) $|\{t' : f_{i+1}(t') = w\}| \leq b(w)$, and (d) $|\{t' : f_i(t') = v \wedge f_{i+1}(t') = w\}| \leq c(v,w)$.

Measure: The length of the schedule, i.e., l.

Bad News: Not in APX [Clementi and Di Ianni, 1996].

Comment: Reduction from MINIMUM GRAPH COLORING. It remains non-approximable even for layered graphs. The variation in which the vertex-capacity is unbounded and the delay, i.e., the maximum number of times a token is neither in the final destination nor moved, must be minimized is also non-approximable within $|T|^{1-\varepsilon}$ for any ε [Di Ianni, 1996].

SS20. MINIMUM VEHICLE SCHEDULING ON TREE

INSTANCE: Rooted tree $T = \langle V, E, v_0 \rangle$, a forward travel time $f : E \to N$, a backward travel time $b : E \to N$, a release time $r : V \to N$, and an handling time $h : V \to N$.

SOLUTION: A vehicle routing schedule that starts from v_0, visits all nodes of T, returns to v_0, and, for any node v_i, starts processing v_i not before the release time $r(v_i)$, i.e., a permutation π of $1, \ldots, |V|$ and a waiting function w such that, for any i,

$$d(v_0, v_{\pi(1)}) + \sum_{j=1}^{i-1} [w(v_{\pi(j)}) + h(v_{\pi(j)}) + d(v_{\pi(j)}, v_{\pi(j+1)})] \geq r(v_{\pi(i)})$$

where $d(u, v)$ denotes the length of the unique path from u to v.

MEASURE: The total completion time, i.e.,

$$d(v_0, v_{\pi(1)}) + \sum_{j=1}^{n-1} [w(v_{\pi(j)}) + h(v_{\pi(j)}) + d(v_{\pi(j)}, v_{\pi(j+1)})] + w(v_{\pi(n)}) + h(v_{\pi(n)}) + d(v_{\pi(n)}, v_0).$$

Good News: Approximable within 2 [Karuno, Nagamochi, and Ibaraki, 1993].

Mathematical Programming

MP1. MINIMUM $0-1$ PROGRAMMING

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$, nonnegative integer n-vector $c \in N^n$.

SOLUTION: A binary n-vector $x \in \{0, 1\}^n$ such that $Ax \geq b$.

MEASURE: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Bad News: NPO-complete [Orponen and Mannila, 1987].

Comment: Transformation from MINIMUM WEIGHTED SATISFIABILITY. Variation in which $c_i = 1$ for all i is NPOPB-complete and not approximable within $n^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1994b]. Variation in which there are at most k non-zero entries on each row of the matrix with k constant is approximable within k [Hall and Hochbaum, 1986 and Hochbaum, Megiddo, Naor, and Tamir, 1993].

Garey and Johnson: MP1

MP2. MAXIMUM BOUNDED $0-1$ PROGRAMMING

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$, nonnegative binary n-vector $c \in \{0, 1\}^n$.

SOLUTION: A binary n-vector $x \in \{0,1\}^n$ such that $Ax \le b$.

MEASURE: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Bad News: NPOPB-complete [Berman and Schnitger, 1992].

Comment: Transformation from LONGEST PATH WITH FORBIDDEN PAIRS. Not approximable within $n^{1-\varepsilon}$ [Jonsson, 1997] and not within $m^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1995].

Garey and Johnson: MP1

MP3. MAXIMUM PACKING INTEGER PROGRAMMING

INSTANCE: Rational $m \times n$-matrix $A \in [0,1]^{m \cdot n}$, rational m-vector $b \in [1,\infty)^m$, rational n-vector $c \in [0,1]^n$.

SOLUTION: A rational n-vector $x \in [0,1]^n$ such that $Ax \le b$.

MEASURE: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Good News: Approximable within $O(opt(cm^{1/B}/opt)^{B/(B-1)})$ for some $c > 0$, where $B = \min_i b_i$ [Srinivasan, 1995].

Comment: Variation in which $A \in \{0,1\}^{m \cdot n}$ and $b \in Z^+$ is approximable within $O(opt(m^{1/(B+1)}/opt)^{(B+1)/B})$ [Srinivasan, 1995].

MP4. MINIMUM COVERING INTEGER PROGRAMMING

INSTANCE: Rational $m \times n$-matrix $A \in [0,1]^{m \cdot n}$, rational m-vector $b \in [1,\infty)^m$, rational n-vector $c \in [0,1]^n$.

SOLUTION: A rational n-vector $x \in [0,1]^n$ such that $Ax \ge b$.

MEASURE: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Good News: Approximable within $1 + O(\max(\log(mB/opt), \sqrt{\log(mB/opt)/B}))$, where $B = \min_i b_i$ [Srinivasan, 1995].

Comment: Variation in which $A \in \{0,1\}^{m \cdot n}$ and $b \in Z^+$ is approximable within $O(opt(m^{1/(B+1)}/opt)^{(B+1)/B})$ [Srinivasan, 1995].

MP5. MAXIMUM QUADRATIC PROGRAMMING

INSTANCE: Positive integer n, set of linear constraints, given as an $m \times n$-matrix A and an m-vector b, specifying a region $S \subseteq R^n$ by $S = \{x \in [0,1]^n : Ax \le b\}$.

SOLUTION: A multivariate polynomial $f(x_1, \ldots, x_n)$ of total degree at most 2.

MEASURE: The maximum value of f in the region specified by the linear constants, i.e., $\max_{x \in S} f(x)$.

Bad News: Does not admit a μ-approximation for any constant $0 < \mu < 1$ [Bellare and Rogaway, 1995].

Comment: A μ-approximation algorithm finds a solution that differs from the optimal solution by at most the value $\mu\,(\max_{x \in S} f(x) - \min_{x \in S} f(x))$. Variation in which we look for a polynomial f of any degree does not admit a μ-approximation for $\mu = 1 - n^{-\delta}$ for some

$\delta > 0$ [Bellare and Rogaway, 1995]. Note that these problems are known to be solvable in polynomial space but are not known to be in NP.

Garey and Johnson: MP2

MP6. MINIMUM GENERALIZED $0-1$ ASSIGNMENT

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$, and binary $m \times n$-matrix $C \in \{0,1\}^{m \cdot n}$.

SOLUTION: A binary $m \times n$-matrix $X \in \{0,1\}^{m \cdot n}$ such that there is exactly one 1 in each column of X, and $\sum_{j=1}^{n} A_{i,j} X_{i,j} \leq b_i$ for all $i \in [1..m]$.

MEASURE: $\sum_{i=1}^{m} \sum_{j=1}^{n} C_{i,j} X_{i,j}$.

Bad News: Not in APX [Sahni and Gonzalez, 1976].

MP7. MINIMUM QUADRATIC $0-1$ ASSIGNMENT

INSTANCE: Nonnegative integer $n \times n$-matrix $C \in N^{n \cdot n}$, nonnegative integer $m \times m$-matrix $D \in N^{m \cdot m}$.

SOLUTION: Binary $n \times m$-matrix $X \in \{0,1\}^{n \cdot m}$ such that there is at most one 1 in each row of X and exactly one 1 in each column of X.

MEASURE: $\sum_{\substack{i,j=1 \\ i \neq j}}^{n} \sum_{\substack{k,l=1 \\ k \neq l}}^{m} C_{i,j} D_{k,l} X_{i,k} X_{j,l}$.

Bad News: Not in APX [Sahni and Gonzalez, 1976].

Comment: Not in APX even if D satisfies the triangle inequality [Queyranne, 1986].

MP8. MINIMUM PLANAR RECORD PACKING

INSTANCE: Collection C of n records, for each record $c \in C$ a probability $p(c)$ such that $0 \leq p(c) \leq 1$.

SOLUTION: For each record $c \in C$ a placement $z(c)$ in the plane, given as integer coordinates, such that all records are placed on different points in the plane.

MEASURE: $\sum_{c_1 \in C} \sum_{c_2 \in C} p(c_1) p(c_2) d(z(c_1), z(c_2))$, where $d(z(c_1), z(c_2))$ is the discretized Euclidean distance between the points $z(c_1)$ and $z(c_2)$.

Good News: Approximable with an absolute error guarantee of $\lfloor 4\sqrt{2} + 8\sqrt{\pi} \rfloor$, that is, one can in polynomial time find a solution with objective function value at most $opt + \lfloor 4\sqrt{2} + 8\sqrt{\pi} \rfloor$ [Karp, McKellar, and Wong, 1975].

MP9. MINIMUM RELEVANT VARIABLES IN LINEAR SYSTEM

INSTANCE: Integer $m \times n$-matrix $A \in Z^{m \cdot n}$, integer m-vector $b \in Z^m$.

SOLUTION: A rational n-vector $x \in Q^n$ such that $Ax = b$.

MEASURE: The number of non-zero elements in x.

Bad News: Not in APX [Amaldi and Kann, 1994].
Comment: Not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP⊂QP [Amaldi and Kann, 1994]. The above nonapproximability results are still true for the variation in which the solutions are restricted by $Ax \geq b$ instead of $Ax = b$. Variation in which the solution vector is restricted to contain binary numbers is NPOPB-complete and is not approximable within $n^{0.5-\varepsilon}$ for any $\varepsilon > 0$ [Amaldi and Kann, 1994]. The complementary maximization problem, where the number of zero elements in the solution is to be maximized, and the solution vector is restricted to contain binary numbers, is NPOPB-complete and is not approximable within $(n+m)^{1/3-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1995].
Garey and Johnson: MP5

MP10. MAXIMUM SATISFYING LINEAR SUBSYSTEM

INSTANCE: System $Ax = b$ of linear equations, where A is an integer $m \times n$-matrix, and b is an integer m-vector.
SOLUTION: A rational n-vector $x \in Q^n$.
MEASURE: The number of equations that are satisfied by x.

Good News: Approximable within $O(m/\log m)$ [Halldórsson, 1995b].
Bad News: Not approximable within m^{ε} for some $\varepsilon > 0$ [Amaldi and Kann, 1995].
Comment: For any prime $q \geq 2$ the problem over GF$[q]$ is approximable within q, but is not approximable within $q - \varepsilon$ for any $\varepsilon > 0$, even if the number of variables in each equation is exactly three. The problem over GF[2] with two variables in each equation is approximable within 1.383 but is not approximable within 1.0909 [Håstad, 1997]. The variation where the system consists of relations ($>$ or $\geq$) is APX-complete and approximable within 2 [Amaldi and Kann, 1995]. If the variables are restricted to assume only binary values, the problem is harder to approximate than MAXIMUM INDEPENDENT SET. Approximability results for even more variants of the problem can be found in [Amaldi and Kann, 1995].

MP11. MINIMUM UNSATISFYING LINEAR SUBSYSTEM

INSTANCE: System $Ax = b$ of linear equations, where A is an integer $m \times n$-matrix, and b is an integer m-vector.
SOLUTION: A rational n-vector $x \in Q^n$.
MEASURE: The number of equations that are *not* satisfied by x.

Bad News: Not in APX [Arora, Babai, Stern, and Sweedyk, 1993].
Comment: Not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP⊂QP [Arora, Babai, Stern, and Sweedyk, 1994]. If the system consists of relations ($>$ or $\geq$) the problem is even harder to approximate; there is a transformation from MINIMUM DOMINATING SET to this problem. If the variables are restricted to assume only binary values the problem is NPOPB-complete both for equations and relations, and is not approximable within $n^{1-\varepsilon}$ for any $\varepsilon > 0$. Approximability results for even more variants of the problem can be found in [Amaldi and Kann, 1994].

MP12. Maximum Hyperplane Consistency

Instance: Finite sets P and N of integer n-vectors. P consists of positive examples and N of negative examples.
Solution: A hyperplane specified by a normal vector $w \in Q^n$ and a bias w_0.
Measure: The number of examples that are consistent with respect to the hyperplane, i.e., $|\{x \in P : wx > w_0\}| + |\{x \in N : wx < w_0\}|$.

Good News: Approximable within 2 [Amaldi and Kann, 1995].
Bad News: Apx-complete [Amaldi and Kann, 1995].
Comment: Variation in which only one type of misclassification, either positive or negative, is allowed is not approximable within n^ε for some $\varepsilon > 0$ [Amaldi and Kann, 1995]. The complementary minimization problem, where the number of misclassifications is to be minimized, is not in Apx unless P=Np, and is not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless Np⊂Qp [Arora, Babai, Stern, and Sweedyk, 1994] and [Amaldi and Kann, 1994].
Garey and Johnson: Similar to MP6

MP13. Maximum Knapsack

Instance: Finite set U, for each $u \in U$ a size $s(u) \in Z^+$ and a value $v(u) \in Z^+$, a positive integer $B \in Z^+$.
Solution: A subset $U' \subseteq U$ such that $\sum_{u \in U'} s(u) \le B$.
Measure: Total weight of the chosen elements, i.e., $\sum_{u \in U'} v(u)$.

Good News: Admits an Fptas [Ibarra and Kim, 1975].
Comment: The special case when $s(u) = v(u)$ for all $u \in U$ is called *Maximum Subset Sum.* The corresponding minimization problem where $\sum_{u \in U'} s(u) \ge B$ also admits an Fptas, as well as several other variations of the knapsack problem [Gens and Levner, 1979].
Garey and Johnson: MP9

MP14. Maximum Integer m-Dimensional Knapsack

Instance: Nonnegative integer $m \times n$-matrix, $A \in N^{m \cdot n}$, nonnegative integer m-vector $b \in N^m$, nonnegative integer n-vector $c \in N^n$. m is a constant, $m \in Z^+$.
Solution: Nonnegative integer n-vector $x \in N^n$ such that $Ax \le b$.
Measure: The scalar product of c and x, i.e., $\sum_{i=1}^{n} c_i x_i$.

Good News: Admits a Ptas [Chandra, Hirschberg, and Wong, 1976].
Garey and Johnson: Generalization of MP10

MP15. Maximum Integer k-Choice Knapsack

Instance: Nonnegative integer $n \times k$-matrices $A, C \in N^{n \cdot k}$, nonnegative integer $b \in N$.

SOLUTION: Nonnegative integer vector $x \in N^n$, function $f : [1..n] \rightarrow [1..k]$ such that $\sum_{i=1}^{n} a_{i,f(i)} x_i \leq b$.

MEASURE: $\sum_{i=1}^{n} c_{i,f(i)} x_i$.

Good News: Admits an FPTAS [Chandra, Hirschberg, and Wong, 1976].

Comment: The corresponding minimization problem, where $\sum_{i=1}^{n} a_{i,f(i)} x_i \geq b$ also admits an FPTAS as well as the variation in which there does not need to be a chosen element for each i [Gens and Levner, 1979].

Garey and Johnson: Similar to MP11

MP16. NEAREST LATTICE VECTOR

INSTANCE: Lattice basis $\{b_1, \ldots, b_m\}$ where $b_i \in Z^k$, a point $b_0 \in Q^k$, and a positive integer p.
SOLUTION: A vector b in the lattice, where $b \neq b_0$.
MEASURE: The distance between b_0 and b in the ℓ_p norm.

Bad News: Not in APX [Arora, Babai, Stern, and Sweedyk, 1993].

Comment: Not approximable within $2^{\log^{1-\varepsilon} mp}$ for any $\varepsilon > 0$ unless NP$\subset$QP [Arora, Babai, Stern, and Sweedyk, 1994]. The special case where b_0 is the zero vector and $p = \infty$ is not approximable within $2^{\log^{0.5-\varepsilon} mp}$ for any $\varepsilon > 0$ unless NP$\subset$QP [Arora, Babai, Stern, and Sweedyk, 1993].

MP17. MINIMUM BLOCK-ANGULAR CONVEX PROGRAMMING

INSTANCE: K disjoint convex compact sets B^k called blocks, M nonnegative continuous convex functions $f_m^k : B^k \rightarrow R$.
SOLUTION: A positive number λ such that

$$\sum_{k=1}^{K} f_m^k(x^k) \leq \lambda \text{ for } 1 \leq m \leq M, \text{ and } x^k \in B^k \text{ for } 1 \leq k \leq K.$$

MEASURE: λ.

Good News: Admits an FPTAS [Grigoriadis and Khachiyan, 1994].

Algebra and Number Theory

Solvability of Equations

AN1. MAXIMUM SATISFIABILITY OF QUADRATIC EQUATIONS OVER GF$[q]$

INSTANCE: Prime number q, set $P = \{p_1(x), p_2(x), \ldots, p_m(x)\}$ of polynomials of degree at most 2 over GF$[q]$ in n variables. The polynomials may not contain any monomial ${x_i}^2$ for any i.

SOLUTION: A subset $P' \subseteq P$ of the polynomials such that there is a root common to all polynomials in P'.
MEASURE: Cardinality of the subset, i.e., $|P'|$.

Good News: Approximable within $q^2/(q-1)$ [Håstad, Phillips, and Safra, 1993].
Bad News: Not approximable within $q-\varepsilon$ for any ε [Håstad, Phillips, and Safra, 1993].
Comment: Over the rationals or over the reals the problem is not approximable within $n^{1-\varepsilon}$ for any $\varepsilon > 0$ [Håstad, Phillips, and Safra, 1993]. For linear polynomials the problem is not approximable within q^{ε} for some $\varepsilon > 0$ [Amaldi and Kann, 1995].
Garey and Johnson: Similar to AN9

Games and Puzzles

GP1. MINIMUM GRAPH MOTION PLANNING

INSTANCE: Graph $G = \langle V, E \rangle$, start vertex $s \in V$ where the robot is initially placed, goal vertex $t \in V$, and a subset $W \subseteq V$ of vertices where the obstacles are initially placed.
SOLUTION: A motion scheme for the robot and the obstacles. In each time step either the robot or one obstacle may be moved to a neighbouring vertex that is not occupied by the robot or an obstacle.
MEASURE: The number of steps until the robot has been moved to the goal vertex t.

Good News: Approximable within $O(\sqrt{|V|})$ [Papadimitriou, Raghavan, Sudan, and Tamaki, 1994].
Bad News: APX-complete [Papadimitriou, Raghavan, Sudan, and Tamaki, 1994].
Comment: Approximable within $O(l_{\max}/l_{\min})$, where $l_{\max}$ and $l_{\min}$ are the lengths of the longest and shortest paths of degree-2 vertices in G.

GP2. MINIMUM TRAVEL ROBOT LOCALIZATION

INSTANCE: A simple polygon P and the visibility polygon V of the location of a robot inside P.
SOLUTION: A motion scheme for the robot to determine its exact location.
MEASURE: The distance the robot travels according to the motion scheme.

Good News: Approximable within $|H| - 1$ where H is the set of points in P with visibility polygon V [Dudek, Romanik, and Whitesides, 1994].

Logic

Propositional Logic

LO1. MAXIMUM SATISFIABILITY

INSTANCE: Set U of variables, collection C of disjunctive clauses of literals, where a literal is a variable or a negated variable in U.
SOLUTION: A truth assignment for U.
MEASURE: Number of clauses satisfied by the truth assignment.

Good News: Approximable within 1.3193 [Goemans and Williamson, 1995b].
Bad News: APX-complete [Papadimitriou and Yannakakis, 1991].
Comment: Variation in which each clause has a nonnegative weight and the objective is to maximize the total weight of the satisfied clauses is also approximable within 1.3193 [Goemans and Williamson, 1995b]. Generalization in which each clause is a disjunction of conjunctions of literals and each conjunction consists of at most k literals, where k is a positive constant, is still APX-complete [Papadimitriou and Yannakakis, 1991]. Admits a PTAS for 'planar' instances [Khanna and Motwani, 1996]. The corresponding minimization problem MINIMUM SATISFIABILITY is approximable within 2 [Bertsimas, Teo, and Vohra, 1996].
Garey and Johnson: LO1

LO2. MAXIMUM k-SATISFIABILITY

INSTANCE: Set U of variables, collection C of disjunctive clauses of at most k literals, where a literal is a variable or a negated variable in U. k is a constant, $k \geq 2$.
SOLUTION: A truth assignment for U.
MEASURE: Number of clauses satisfied by the truth assignment.

Good News: Approximable within $1/(1-2^{-k})$ if every clause consists of exactly k literals [Johnson, 1974].
Bad News: APX-complete [Papadimitriou and Yannakakis, 1991].
Comment: MAXIMUM 3-SATISFIABILITY is approximable within 1.249 [Trevisan, Sorkin, Sudan, and Williamson, 1996]. MAXIMUM k-SATISFIABILITY is not approximable within $1/(1-2^{-k}) - \varepsilon$ for any $\varepsilon > 0$ and $k \geq 3$, even if every clause consists of exactly k literals [Håstad, 1997].
MAXIMUM 2-SATISFIABILITY is approximable within 1.0741 [Feige and Goemans, 1995], and is not approximable within 1.0476 [Håstad, 1997]. Variation in which the number of occurrences of any literal is bounded by a constant B for $B \geq 3$ is still APX-complete [Papadimitriou and Yannakakis, 1991]. Admits a PTAS if $|C| = \Theta(|U|^k)$ [Arora, Karger, and Karpinski, 1995]. Variation in which each clause is a Horn clause, i.e., contains at most one nonnegated variable, is APX-complete, even for $k = 2$ [Kohli, Krishnamurti, and Mirchandani, 1994].
Garey and Johnson: LO2 and LO5

LO3. MINIMUM k-SATISFIABILITY

INSTANCE: Set U of variables, collection C of disjunctive clauses of at most k literals, where a literal is a variable or a negated variable in U. k is a constant, $k \geq 2$.
SOLUTION: A truth assignment for U.
MEASURE: Number of clauses satisfied by the truth assignment.

Good News: Approximable within $2(1 - 1/2^k)$ [Bertsimas, Teo, and Vohra, 1996].
Bad News: APX-complete for every $k \geq 2$ [Kohli, Krishnamurti, and Mirchandani, 1994].
Comment: Transformation from MAXIMUM 2-SATISFIABILITY. Variation in which each clause is a Horn clause, i.e., contains at most one nonnegated variable, is APX-complete, even for $k = 2$ [Kohli, Krishnamurti, and Mirchandani, 1994].
Garey and Johnson: LO2

LO4. Maximum Not-All-Equal 3-Satisfiability

Instance: Set U of variables, collection C of disjunctive clauses of at most 3 literals, where a literal is a variable or a negated variable in U.
Solution: A truth assignment for U and a subset $C' \subseteq C$ of the clauses such that each clause in C' has at least one true literal and at least one false literal.
Measure: $|C'|$.

Good News: Approximable within 1.138 [Kann, Lagergren, and Panconesi, 1996].
Bad News: Apx-complete [Papadimitriou and Yannakakis, 1991].
Comment: Transformation from Maximum 2-Satisfiability. Not approximable within 1.013 [Kann, Lagergren, and Panconesi, 1996].
Garey and Johnson: LO3

LO5. Minimum 3DNF Satisfiability

Instance: Set U of variables, collection C of conjunctive clauses of at most three literals, where a literal is a variable or a negated variable in U.
Solution: A truth assignment for U.
Measure: Number of clauses satisfied by the truth assignment.

Bad News: Not in Apx [Kolaitis and Thakur, 1994].
Garey and Johnson: LO8

LO6. Maximum Distinguished Ones

Instance: Disjoint sets X, Z of variables, collection C of disjunctive clauses of at most 3 literals, where a literal is a variable or a negated variable in $X \cup Z$.
Solution: Truth assignment for X and Z that satisfies every clause in C.
Measure: The number of Z variables that are set to true in the assignment.

Bad News: Npopb-complete [Kann, 1992b].
Comment: Transformation from Maximum Number of Satisfiable Formulas [Panconesi and Ranjan, 1993]. Not approximable within $(|X| + |Z|)^{1-\varepsilon}$ for any $\varepsilon > 0$ [Jonsson, 1997]. Maximum Ones, the variation in which all variables are distinguished, i.e. $|X| = \emptyset$, is also Npopb-complete [Kann, 1992b], and is not approximable within $|Z|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Jonsson, 1997]. Maximum Weighted Satisfiability, the weighted version, in which every variable is assigned a nonnegative weight, is Npo-complete [Ausiello, D'Atri, and Protasi, 1981].

LO7. Minimum Distinguished Ones

Instance: Disjoint sets X, Z of variables, collection C of disjunctive clauses of at most 3 literals, where a literal is a variable or a negated variable in $X \cup Z$.
Solution: Truth assignment for X and Z that satisfies every clause in C.
Measure: The number of Z variables that are set to true in the assignment.

Bad News: Npopb-complete [Kann, 1994b].

Comment: Transformation from Minimum Independent Dominating Set. Not approximable within $(|X| + |Z|)^{1-\varepsilon}$ for any $\varepsilon > 0$ [Jonsson, 1997]. Minimum Ones, the variation in which all variables are distinguished, i.e. $|X| = \emptyset$, is also Npopb-complete [Kann, 1994b], and is not approximable within $|Z|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Jonsson, 1997]. Minimum Ones for clauses of 2 literals is approximable within 2 [Gusfield and Pitt, 1992]. Minimum Weighted Satisfiability, the weighted version, in which every variable is assigned a nonnegative weight, is Npo-complete [Orponen and Mannila, 1987]. Variations corresponding to three- and four-valued logics have been also considered [Errico and Rosati, 1995].

LO8. Maximum Weighted Satisfiability with Bound

Instance: Set U of variables, boolean expression F over U, a nonnegative bound $B \in N$, for each variable $u \in U$ a weight $w(u) \in N$ such that $B \le \sum_{u \in U} w(u) \le 2B$.

Solution: A truth assignment for U, i.e., a subset $U' \subseteq U$ such that the variables in U' are set to true and the variables in $U - U'$ are set to false.

Measure: $\sum_{v \in U'} w(v)$ if the truth assignment satisfies the boolean expression F and B otherwise.

Good News: Approximable within 2 [Crescenzi and Panconesi, 1991].

Bad News: Apx-complete [Crescenzi and Panconesi, 1991].

Comment: Variation in which $\sum_{u \in U} w(u) \le (1 + 1/(|U| - 1))\ B$ is Ptas-complete [Crescenzi and Panconesi, 1991].

LO9. Maximum Number of Satisfiable Formulas

Instance: Set U of variables, collection C of 3CNF formulas.

Solution: A truth assignment for U.

Measure: Number of formulas satisfied by the truth assignment.

Bad News: Npopb-complete [Kann, 1992b].

Comment: Transformation from Longest Induced Path. Not approximable within $|C|$.

LO10. Minimum Number of Satisfiable Formulas

Instance: Set U of variables, collection C of 3CNF formulas.

Solution: A truth assignment for U.

Measure: Number of clauses satisfied by the truth assignment.

Bad News: Npopb-complete [Kann, 1994b].

Comment: Transformation from Minimum Distinguished Ones. Not approximable within $|C|^{1-\varepsilon}$ for any $\varepsilon > 0$ [Kann, 1994b].

LO11. Minimum Equivalence Deletion

Instance: Set U of variables, collection C of equivalences, i.e., pairs of literals over U.

Solution: A truth assignment for U.

MEASURE: Number of equivalences that are not satisfied by the truth assignment.

Good News: Approximable within $O(\log |U|)$ [Garg, Vazirani, and Yannakakis, 1996].
Bad News: APX-hard [Garg, Vazirani, and Yannakakis, 1996].
Comment: The complementary maximization problem is approximable within 1.138 [Kann, —].

LO12. MAXIMUM k-CONSTRAINT SATISFACTION

INSTANCE: Set U of variables, collection C of conjunctive clauses of at most k literals, where a literal is a variable or a negated variable in U, and k is a constant, $k \geq 2$.
SOLUTION: A truth assignment for U.
MEASURE: Number of clauses satisfied by the truth assignment.

Good News: Approximable within 2^{k-1} [Trevisan, 1996].
Bad News: APX-complete [Berman and Schnitger, 1992].
Comment: Transformation from MAXIMUM 2-SATISFIABILITY. For large enough k, it is not approximable within $2^{0.09k}$ [Trevisan, 1996]. Not in APX when $k = \log |C|$ [Verbitsky, 1995].

Miscellaneous

LO13. MAXIMUM HORN CORE

INSTANCE: Set M of truth assignments on n variables.
SOLUTION: A Horn core of M, i.e., a subset $M' \subseteq M$ such that M' is equal to the set of truth assignments satisfying a Horn boolean formula.
MEASURE: The cardinality of the core, i.e., $|M'|$.

Bad News: Not in APX [Kavvadias, Papadimitriou, and Sideri, 1993].

Automata and Language Theory

Automata Theory

AL1. MINIMUM CONSISTENT FINITE AUTOMATON

INSTANCE: Two finite sets P, N of binary strings.
SOLUTION: A deterministic finite automaton $\langle Q, \{0,1\}, \delta, q_0, F\rangle$ accepting all strings in P and rejecting all strings in N.
MEASURE: Number of states in the automaton.

Bad News: Not approximable within $2-\varepsilon$ for any $\varepsilon > 0$ [Simon, 1990].
Comment: Transformation from MINIMUM GRAPH COLORING. Not approximable within $(|P|+|N|)^{1/14-\varepsilon}$ for any $\varepsilon > 0$ [Pitt and Warmuth, 1993].
Garey and Johnson: AL8

AL2. LONGEST COMPUTATION

INSTANCE: Nondeterministic Turing machine M, binary input string x.
SOLUTION: Nondeterministic guess string c produced by M on input x.
MEASURE: The length of the shortest of the strings c and x, i.e., $\min(|c|, |x|)$.

Bad News: NPOPB-complete [Berman and Schnitger, 1992].
Comment: Variation in which the Turing machine is oblivious is also NPOPB-complete.

AL3. SHORTEST COMPUTATION

INSTANCE: Nondeterministic Turing machine M, binary input string x.
SOLUTION: Nondeterministic guess string c produced by M on input x.
MEASURE: The length of the shortest of the strings c and x, i.e., $\min(|c|, |x|)$.

Bad News: NPOPB-complete [Kann, 1994b].
Comment: Variation in which the Turing machine is oblivious is also NPOPB-complete.

Formal Languages

AL4. MINIMUM LOCALLY TESTABLE AUTOMATON ORDER

INSTANCE: A locally testable language L, i.e., a language L such that, for some positive integer j, whether or not a string x is in the language depends on (a) the prefix and suffix of x of length $j-1$, and (b) the set of substrings of x of length j.
SOLUTION: An order j, i.e., a positive integer j witnessing the local testability of L.
MEASURE: The value of the order, i.e., j.

Good News: Admits a PTAS [Kim and McNaughton, 1993].

Miscellaneous

AL5. MINIMUM PERMUTATION GROUP BASE

INSTANCE: Permutation group on n letters.
SOLUTION: A base for G, i.e., a sequence of points $b_1, \ldots, b_k$ such that the only element in G fixing all of the b_i is the identity.
MEASURE: The size of the base, i.e., k.

Good News: Approximable within $\log \log n$ [Blaha, 1992].

Program Optimization

Code Generation

PO1. MINIMUM REGISTER SUFFICIENCY

INSTANCE: Directed acyclic graph $G = \langle V, E \rangle$.
SOLUTION: Computation for G that uses k registers, i.e., an ordering $v_1, \ldots, v_n$ of the vertices in V and a sequence $S_0, \ldots, S_n$ of subsets of V, each satisfying $|S_i| \le k$, such that S_0 is empty, S_n contains all vertices with in-degree 0 in G, and, for $1 \le i \le n$, $v_i \in S_i$, $S_i - \{v_i\} \subseteq S_{i-1}$, and S_{i-1} contains all vertices u for which $(v_i, u) \in A$.
MEASURE: Number of registers, i.e., k.

Good News: Approximable within $O(\log^2 n)$ [Klein, Agrawal, Ravi, and Rao, 1990].
Garey and Johnson: PO1

Miscellaneous

MS1. MAXIMUM BETWEENNESS

INSTANCE: Finite set A, collection C of ordered triples (a, b, c) of distinct elements from A.
SOLUTION: A one-to-one function $f : A \to [1..|A|]$.
MEASURE: Number of triples where either $f(a) < f(b) < f(c)$ or $f(c) < f(b) < f(a)$.

Good News: Approximable within 2 [Chor and Sudan, 1995].
Bad News: APX-complete [Chor and Sudan, 1995].
Comment: Admits a PTAS if $|C| = \Theta(|A|^3)$ [Arora, Frieze, and Kaplan, 1996].
Garey and Johnson: MS1

MS2. NEAREST CODEWORD

INSTANCE: Linear binary code C of length n and a string x of length n.
SOLUTION: A codeword y of C.
MEASURE: The Hamming distance between x and y, i.e., $d(x, y)$.

Bad News: Not in APX [Arora, Babai, Stern, and Sweedyk, 1993].
Comment: Not approximable within $2^{\log^{1-\varepsilon} n}$ for any $\varepsilon > 0$ unless NP⊂QP [Arora, Babai, Stern, and Sweedyk, 1994]. The complementary maximization problem, where the number of bits that agree between x and y is to be maximized, does not admit a PTAS [Petrank, 1992].

MS3. MINIMUM TREE ALIGNMENT

INSTANCE: Set S of sequences over an alphabet Σ, a score scheme $\mu : \Sigma \cup \{\Delta\} \times \Sigma \cup \{\Delta\} \to N$ satisfying the triangle inequality, and a tree T of bounded degree whose leafs are labeled with the sequences S.
SOLUTION: A sequence labeling of the interior nodes of the tree T.
MEASURE: The total alignment cost of the labeled tree, i.e., the sum over all edges (x, y) in the tree of the edit distance $d(x, y)$ between the labels of the endpoints of the edge. The edit distance is the minimum alignment cost $\sum_i \mu(x'[i], y'[i])$ over all possible alignments x' and y' of x and y. An alignment is obtained by inserting spaces (denoted by Δ) into the original sequence, either between two characters or in the beginning or at the end. x' and y' must have the same length.

Good News: Admits a PTAS [Jiang, Lawler, and Wang, 1994].
Comment: Variation in which the tree is not given as an input and the objective is to find the tree with the minimum total alignment cost is called *Minimum Generalized Tree Alignment* or *Minimum Evolutionary Tree* and is APX-hard [Jiang, Lawler, and Wang, 1994].

MS4. MINIMUM ATTRACTION RADIUS FOR BINARY HOPFIELD NET

INSTANCE: An n-node synchronous binary Hopfield network and a stable initial vector of states $u \in \{-1, 1\}^n$. A binary Hopfield network is a complete graph where each edge has an integer weight $w(v_i, v_j)$ and each vertex has an integer threshold value $t(v_i)$. At each time step t each vertex v_i has a state $x(t, v_i)$. $x(0, v_i)$ is given by u and $x(t+1, v_i) = \text{sgn}\left(\sum_{j=1}^{n} w(v_i, v_j) - t_i\right)$ where sgn is the sign function. An initial vector of states is stable if $x(t, v_i)$ eventually converges for all i.
SOLUTION: An initial vector of states v that either converges to a different vector than u or is not stable.
MEASURE: The Hamming distance between u and v. If v is the vector nearest to u that does not converge to the same vector as u, then this distance is the attraction radius.

Bad News: Not approximable within $n^{1-\varepsilon}$ for any $\varepsilon > 0$ [Floréen and Orponen, 1993].
Comment: Transformation from MINIMUM INDEPENDENT DOMINATING SET.

MS5. MAXIMUM CHANNEL ASSIGNMENT

INSTANCE: Net of hexagonal cells in which n cells C_i are assigned a positive load r_i, an interference radius r, and m channels F_i.
SOLUTION: A channel assignment A, i.e., a multivalued function A assigning a set of cells to a channel such that if $C_i, C_j \in A(F_k)$ then the distance between C_i and C_j is greater than $2r$.
MEASURE: The number of satisfied request, i.e., $\sum_i \min\{r_i, |\{F_j : C_i \in F_j\}|\}$.

Good News: Approximable within $1/(1 - e^{-1})$ [Simon, 1989].
Comment: Admits a PTAS if the number of channels is fixed. Similar results hold in the case in which each cell has a set of forbidden channels.

MS6. MINIMUM k-LINK PATH IN A POLYGON

INSTANCE: Polygon P with n integer-coordinates vertices and two points s and t in P.
SOLUTION: A k-link path between s and t, i.e., a sequence $p_0, \ldots, p_h$ of points inside P with $h \le k$ such that $p_0 = s$, $p_k = t$, and, for all i with $0 \le i < h$, the segment between p_i and p_{i+1} is inside P.
MEASURE: The Euclidean length of the path, i.e., $\sum_{i=0}^{h-1} d(p_i, p_{i+1})$ where d denotes the Euclidean distance.

Good News: Admits an FPTAS [Mitchell, Piatko, and Arkin, 1992].

MS7. MINIMUM SIZE ULTRAMETRIC TREE

INSTANCE: $n \times n$ matrix M of positive integers.

SOLUTION: An ultrametric tree, i.e., an edge-weighted tree $T(V, E)$ with n leaves such that, for any pair of leaves i and j, $d_{ij}^T \geq M[i,j]$ where d_{ij}^T denotes the sum of the weights in the path between i and j.
MEASURE: The size of the tree, i.e., $\sum_{e \in E} w(e)$ where $w(e)$ denotes the weight of edge e.

Bad News: Not approximable within n^ε for some $\varepsilon > 0$ [Farach, Kannan, and Warnow, 1993].
Comment: Transformation from graph coloring.

MS8. MINIMUM PARTITION OF RECTANGLE WITH INTERIOR POINTS

INSTANCE: Rectangle R and finite set P of points located inside R.
SOLUTION: A set of line segments that partition R into rectangles such that every point in P is on the boundary of some rectangle.
MEASURE: The total length of the introduced line segments.

Good News: Approximable within 1.75 [Gonzalez and Zheng, 1989].
Comment: Variation in which R is a rectilinear polygon is approximable within 4 [Gonzalez and Zheng, 1990].

MS9. MINIMUM SORTING BY REVERSALS

INSTANCE: Permutation π of the numbers 1 to n.
SOLUTION: A sequence $\rho_1, \rho_2, \ldots, \rho_t$ of reversals of intervals such that $\pi \cdot \rho_1 \cdot \rho_2 \cdots \rho_t$ is the identity permutation. A reversal of an interval $[i, j]$ is the permutation $(1, 2, \ldots, i-1, j, j-1, \ldots, i+1, i, j+1, \ldots, n)$.
MEASURE: The number of reversals, i.e., t.

Good News: Approximable within 7/4 [Bafna and Pevzner, 1993].
Comment: Minimum Sorting by Transpositions, the problem where transpositions are used instead of reversals, is approximable within 3/2. A transposition $\rho(i, j, k)$ where $i < j < k$ is the permutation $(1, 2, \ldots, i-1, j, \ldots, k-1, i \ldots, j-1, k, \ldots, n)$ [Bafna and Pevzner, 1995].

MS10. MAXIMUM COMPATIBLE BINARY CONSTRAINT SATISFACTION

INSTANCE: Set $\{x_1, \ldots, x_n\}$ of variables taking values from the sets $D_1, \ldots, D_n$ respectively, binary compatibility constraints $C(x_i, x_j) \subseteq D_i \times D_j$ for $1 \leq i < j \leq n$.
SOLUTION: A compatible variable assignment to a subset of the variables such that for all pairs (x_i, x_j) of variables in the subset, the corresponding values $(d_i, d_j) \in C(x_i, x_j)$.
MEASURE: The number of variables in the compatible variable assignment.

Bad News: Not approximable within $(\sum_i |D_i|)^{1/4-\varepsilon}$ for any $\varepsilon > 0$ [Jagota, 1993] and [Bellare, Goldreich, and Sudan, 1995].
Comment: Equivalent to MAXIMUM CLIQUE under PTAS-reduction [Jagota, 1993].

MS11. SHORTEST PATH MOTION IN 3 DIMENSIONS

INSTANCE: Two points s, t and several polyhedral obstacles in three-dimensional Euclidean space.

SOLUTION: A path from s to t that avoids the obstacles.
MEASURE: The total length of the path in Euclidean metric.

Good News: Admits an FPTAS [Papadimitriou, 1985] and [Choi, Sellen, and Yap, 1994].

MS12. MAXIMUM MAP LABELING

INSTANCE: A set of n points in the plane.
SOLUTION: A collection of n axis-aligned equal-sized squares (labels) such that (a) every point is the corner of exactly one square, and (b) all squares are pairwise disjoint.
MEASURE: The area of the squares.

Good News: Approximable within 2 [Wagner and Wolff, 1995].
Bad News: Non approximable within $2-\varepsilon$ for any ε [Formann and Wagner, 1991].
Comment: The generalized problem, where equal-sized rectangles or ellipses are allowed as labels, and where the labels may have any orientation and may have any point on the boundary at the point in the plane, is in APX. The generalized problem is approximable within 14 when the labels are squares and within 15 when the labels are circles [Doddi, Marathe, Mirzaian, Moret, and Zhu, 1997].

MS13. MINIMUM PHYLOGENETIC TREE DISTANCE

INSTANCE: A 3-degree (binary) tree $T = \langle V, E \rangle$ with unique labels on the leaves and with edge weights $w : E \to N$. A 3-degree tree T' with the same set of unique labels as T.
SOLUTION: A sequence of edges $e_1, \ldots, e_m$ representing nearest neighbour interchanges transforming the tree T into T' (so that the labels match). A nearest neighbour interchange of $e_i = \{u, v\}$ is a transformation where one of the two subtrees adjacent to u is exchanged with one of the two subtrees adjacent to v.
MEASURE: The cost of the sequence of interchanges, i.e., $\sum_{1 \le i \le m} w(e_i)$.

Good News: Approximable within $6 + 6\log n$ where n is the number of leaves [Dasgupta, He, Jiang, Li, Tromp, and Zhang, 1997].
Comment: The variation when another type of transformation called *linear-cost subtree transfer* is used is approximable within 2 [Dasgupta, He, Jiang, Li, Tromp, and Zhang, 1997].

MS14. MINIMUM FREQUENCY ALLOCATION

INSTANCE: Graph $G = \langle V, E \rangle$, and, for each node u, a finite set $L(u) \subset Z^+$ of available frequencies and two integers M_u.
SOLUTION: A frequency allocation for G, i.e., for each node u, a subset $S(u)$ of $L(u)$ such that, for all $u \in V$, $M_u \ge |S(u)|$, and, for all $(u, v) \in E$, $S(u) \cap S(v) = \emptyset$.
MEASURE: The stability number of the allocation, i.e., $\sum_{u \in V} |S(u)|$.

Good News: Approximable within 2 [Malesinska and Panconesi, 1996].
Bad News: APX-complete [Malesinska and Panconesi, 1996].
Comment: Reduction from MAXIMUM 3-SATISFIABILITY.

References

Aggarwal, A., Coppersmith, D., Khanna, S., Motwani, R., and Schieber, B. (1997), "The angular-metric traveling salesman problem", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 221–229. *(ND31)*

Aggarwal, M., and Garg, N. (1994), "A scaling technique for better network design", *Proc. 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 233–239. *(ND9)*

Akutsu, T., and Halldórsson, M. (1994), "On the approximation of largest common point sets and largest common subtrees", *Proc. 5th Ann. Int. Symp. on Algorithms and Computation*, Lecture Notes in Comput. Sci. 834, Springer-Verlag, 405–413. *(SR7, SR8)*

Alimonti, P., and Kann, V. (1997), "Hardness of approximating problems on cubic graphs", *Proc. 3rd Italian Conf. on Algorithms and Complexity*, Lecture Notes in Comput. Sci. 1203, Springer-Verlag, 288–298. *(GT1, ND11)*

Alon, N., Azar, Y., Woeginger, G. J., and Yadid, T. (1997), "Approximation schemes for scheduling", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 493–500. *(SS11)*

Alon, N., Feige, U., Wigderson, A., and Zuckerman, D. (1995), "Derandomized graph products", *Computational Complexity* **5**, 60–75. *(GT21)*

Alon, N., and Kahale, N. (1994), "Approximating the independence number via the θ-function", Unpublished manuscript. *(GT20)*

Alon, N., Yuster, R., and Zwick, U. (1994), "Color-coding: a new method for finding simple paths, cycles and other small subgraphs within large graphs", *Proc. 26th Ann. ACM Symp. on Theory of Comp.*, ACM, 326–335. *(ND39)*

Amaldi, E., and Kann, V. (1994), "On the approximability of removing the smallest number of relations from linear systems to achieve feasibility", Technical Report ORWP-6-94, Department of Mathematics, Swiss Federal Institute of Technology, Lausanne, and Technical Report TRITA-NA-9402, Department of Numerical Analysis and Computing Science, Royal Institute of Technology, Stockholm. *(MP9, MP11, MP12)*

Amaldi, E., and Kann, V. (1995), "The complexity and approximability of finding maximum feasible subsystems of linear relations", *Theoretical Computer Science* **147**, 181–210. *(MP10, MP12, AN1)*

Anily, A., Bramel, J., and Simchi-Levi, D. (1994), "Worst-case analysis of heuristics for the bin-packing problem with general cost structures", *Oper. Res.* **42**, 287–298. *(SR1)*

Anily, S., and Hassin, R. (1992), "The swapping problem", *Networks* **22**, 419–433. *(ND30)*

Arkin, E. M., Chiang, Y., Mitchell, J. S. B., Skiena, S. S., and Yang, T. (1997), "On the maximum scatter TSP", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 211–220. *(ND33)*

Arkin, E. M., Halldórsson, M. M., and Hassin, R. (1993), "Approximating the tree and tour covers of a graph", *Inform. Process. Lett.* **47**, 275–282. *(GT1)*

Arkin, E. M., and Hassin, R. (1992), "Multiple-choice minimum diameter problems", Unpublished manuscript. *(SP4)*

Arkin, E. M., and Hassin, R. (1993), "Approximation algorithms for the geometric covering salesman problem", *Disc. Appl. Math.* , to appear. *(ND30)*

Arkin, E.M., Hassin, R., and Klein, L. (1994), "Restricted delivery problems on a network", Unpublished manuscript. *(ND30)*

Armen, C., and Stein, C. (1994), "A $2\frac{3}{4}$-approximation algorithm for the shortest superstring problem", Technical Report PCS-TR94-214, Department of Computer Science, Dartmouth College, Hanover, New Hampshire. *(SR5)*

Arora, S. (1996), "Polynomial time approximation scheme for euclidean TSP and other geometric problems", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 2–11. *(ND1, ND3, ND7, ND31)*

Arora, S. (1997), "Nearly linear time approximation schemes for Euclidean TSP and other geometric problems", Unpublished manuscript. *(ND7, ND31)*

Arora, S., Babai, L., Stern, J., and Sweedyk, Z. (1993), "The hardness of approximate optima in lattices, codes, and systems of linear equation", *Proc. 34th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 724–733. *(MP11, MP16, MS2)*

Arora, S., Babai, L., Stern, J., and Sweedyk, Z. (1994), "The hardness of approximate optima in lattices, codes, and systems of linear equation", Unpublished manuscript. *(MP11, MP12, MP16, MS2)*

Arora, S., Frieze, A., and Kaplan, H. (1996), "A new rounding procedure for the assignment problem with applications to dense graph arrangement problems", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 21–30. *(GT9, GT40, GT41, MS1)*

Arora, S., Karger, D., and Karpinski, M. (1995), "Polynomial time approximation schemes for dense instances of NP-hard problems", *Proc. 27th Ann. ACM Symp. on Theory of Comp.*, ACM, 284–293. *(GT31, GT32, ND11, ND13, ND16, ND18, ND21, SP3, LO2)*

Aumann, Y., and Rabani, Y. (1995), "Improved bounds for all optical routing", *Proc. 6th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 567–576. *(ND44)*

Aumann, Y., and Rabani, Y. (1996), "An $o(\log k)$ approximate min-cut max-flow theorem and approximation algorithm", *SIAM J. Comp.* , to appear. *(ND20)*

Ausiello, G., D'Atri, A., and Protasi, M. (1980), "Structure preserving reductions among convex optimization problems", *J. Comput. System Sci.* **21**, 136–153. *(GT8, SP2, SP4, SP7)*

Ausiello, G., D'Atri, A., and Protasi, M. (1981), "Lattice theoretic ordering properties for NP-complete optimization problems", *Annales Societatis Mathematicae Polonae* **4**, 83–94. *(LO6)*

Awerbuch, B., Azar, Y., Blum, A., and Vempala, S. (1995), "Improved approximation guarantees for minimum-weight k-trees and prize-collecting salesmen", *Proc. 27th Ann. ACM Symp. on Theory of Comp.*, ACM, 277–283. *(ND7)*

Bafna, V., Berman, P., and Fujito, T. (1994), "Approximating feedback vertex set for undirected graphs within ratio 2", Unpublished manuscript. *(GT8)*

Bafna, V., and Pevzner, P. A. (1993), "Genome rearrangements and sorting by reversals", *Proc. 34th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 148–157. *(MS9)*

Bafna, V., and Pevzner, P. A. (1995), "Sorting permutations by transpositions", *Proc. 6th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 614–621. *(MS9)*

Baker, B. S. (1994), "Approximation algorithms for NP-complete problems on planar graphs", *J. ACM* **41**, 153–180. *(GT1, GT2, GT3, GT10, GT11, GT21)*

Bandelt, H., Crama, Y., and Spieksma, F. C. R. (1991), "Approximation algorithms for multidimensional assignment problems with decomposable costs", Technical Report RRR 33-91, Rutgers Center for Operations Research. *(SP10)*

Bar-Ilan, J., Kortsarz, G., and Peleg, D. (1996), "Generalized submodular cover problems and applications", *Proc. 4th Israel Symp. on Theory of Computing and Systems*, IEEE Computer Society, 110–118. *(ND7)*

Bar-Ilan, J., and Peleg, D. (1991), "Approximation algorithms for selecting network centers", *Algorithms and Data structures*, Lecture Notes in Comput. Sci. 519, Springer-Verlag, 343–354. *(ND48)*

Bar-Yehuda, R., and Even, S. (1985), "A local-ratio theorem for approximating the weighted vertex cover problem", in *Analysis and Design of Algorithms for Combinatorial Problems*, volume 25 of *Annals of Disc. Math.*, Elsevier science publishing company, Amsterdam, 27–46. *(GT1)*

Barvinok, A. I. (1996), "Two algorithmic results for the traveling salesman problem", *Math. Oper. Res.* **21**, 65–84. *(ND31)*

Bellare, M. (1993), "Interactive proofs and approximation: reductions from two provers in one round", *Proc. 2nd Israel Symp. on Theory of Computing and Systems*, IEEE Computer Society, 266–274. *(ND39, ND42, SP11)*

Bellare, M., Goldreich, O., and Sudan, M. (1995), "Free bits, PCPs and non-approximability – towards tight results", *Proc. 36th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 422–431. *(GT5, GT20, SR6, MS10)*

Bellare, M., Goldwasser, S., Lund, C., and Russell, A. (1993), "Efficient probabilistically checkable proofs and applications to approximation", *Proc. 25th Ann. ACM Symp. on Theory of Comp.*, ACM, 294–304. *(GT2, SP4)*

Bellare, M., and Rogaway, P. (1995), "The complexity of approximating a nonlinear program", in , volume 69, , , 429–441. *(MP5)*

Berger, B., and Cowen, L. (1991), "Complexity results and algorithms for $\{<, \leq, =\}$-constrained scheduling", *Proc. Second Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 137–147. *(SS7)*

Berger, B., and Shor, P. W. (1990), "Approximation algorithms for the maximum acyclic subgraph problem", *Proc. First Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 236–243. *(GT9)*

Berman, F., Johnson, D., Leighton, T., Shor, P. W., and Snyder, L. (1990), "Generalized planar matching", *J. Algorithms* **11**, 153–184. *(GT11)*

Berman, P., and Fujito, T. (1995), "Approximating independent sets in degree 3 graphs", *Proc. 4th Workshop on Algorithms and Data Structures*, Lecture Notes in Comput. Sci. 955, Springer-Verlag, 449–460. *(GT1, GT21, SP2)*

Berman, P., and Fürer, M. (1994), "Approximating maximum independent set in bounded degree graphs", *Proc. 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 365–371. *(GT21)*

Berman, P., and Ramaiyer, V. (1994), "Improved approximations for the Steiner tree problem", *J. Algorithms* **17**, 381–408. *(ND8)*

Berman, P., and Schnitger, G. (1992), "On the complexity of approximating the independent set problem", *Inform. and Comput.* **96**, 77–94. *(GT26, GT46, SR6, MP2, LO12, AL2)*

Bern, M., and Plassmann, P. (1989), "The Steiner problem with edge lengths 1 and 2", *Inform. Process. Lett.* **32**, 171–176. *(ND7)*

Bernstein, D., Rodeh, M., and Gertner, I. (1989), "Approximation algorithms for scheduling arithmetic expressions on pipelined machines", *J. Algorithms* **10**, 120–139. *(SS3)*

Bertsimas, D., Teo, C-P., and Vohra, R. (1996), "On dependent randomized rounding algorithms", *Proc. 5th Int. Conf. on Integer Prog. and Combinatorial Optimization*, Lecture Notes in Comput. Sci. 1084, Springer-Verlag, 330–344. *(GT7, LO1, LO3)*

Blaha, K. D. (1992), "Minimum bases for permutation groups: the greedy approximation", *J. Algorithms* **13**, 297–306. *(AL5)*

Blum, A., Chalasani, P., Coppersmith, D., Pulleyblank, B., Raghavan, P., and Sudan, M. (1994), "The minimum latency problem", *Proc. 26th Ann. ACM Symp. on Theory of Comp.*, ACM, 163–171. *(ND30)*

Blum, A., Jiang, T., Li, M., Tromp, J., and Yannakakis, M. (1994), "Linear approximation of shortest superstrings", *J. ACM* **41**, 630–647. *(SR5)*

Blundo, C., De Santis, A., and Vaccaro, U. (1994), "Randomness in distribution protocols", Unpublished manuscript. *(GT22)*

Bodlaender, H. L., Gilbert, J. R., Hafsteinsson, H., and Kloks, T. (1995), "Approximating treewidth, pathwidth, frontsize and shortest elimination tree", *J. Algorithms* **18**, 238–255. *(GT50)*

Bonizzoni, P., Duella, M., and Mauri, G. (1994), "Approximation complexity of longest common subsequence and shortest common supersequence over fixed alphabet", Technical Report 117/94, Dipartimento di Scienze dell'Informazione, Università degli Studi di Milano. *(SR4, SR6)*

Boppana, R., and Halldórsson, M. M. (1992), "Approximating maximum independent sets by excluding subgraphs", *Bit* **32**, 180–196. *(GT20)*

Bui, T. N., and Jones, C. (1992), "Finding good approximate vertex and edge partitions is NP-hard", *Inform. Process. Lett.* **42**, 153–159. *(ND21, ND22)*

Călinescu, G., Fernandes, C. G., Finkler, U., and Karloff, H. (1996), "A better approximation algorithm for finding planar subgraphs", *Proc. 7th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 16–25. *(GT29)*

Chakrabati, S., Phillips, C. A., Schulz, A. S., Shmoys, D. B., Stein, C., and Wein, J. (1996), "Improved scheduling algorithms for minsum criteria", *Proc. 23rd Int. Colloquium on Automata, Languages and Programming*, Lecture Notes in Comput. Sci. 1099, Springer-Verlag, 646–657. *(SS12)*

Chandra, A. K., Hirschberg, D. S., and Wong, C. K. (1976), "Approximate algorithms for some generalized knapsack problems", *Theoretical Computer Science* **3**, 293–304. *(MP14, MP15)*

Chaudhary, A., and Vishwanathan, S. (1997), "Approximation algorithms for the achromatic number", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 558–563. *(GT6)*

Chekuri, C., Motwani, R., Natarajan, B., and Stein, C. (1997), "Approximation techniques for average completion time scheduling", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 609–618. *(SS4, SS12)*

Chen, B. (1993), "A better heuristic for preemptive parallel machine scheduling with batch setup times", *SIAM J. Comp.* **22**, 1303–1318. *(SS9)*

Chen, B. (1994), "Scheduling multiprocessor flow shops", in *Advances in Optimization and Approximation*, Kluwer Academic Publishers, The Netherlands, 1–8. *(SS15)*

Chen, B., Glass, C. A., Potts, C. N., and Strusevich, V. A. (1995), "A new heuristic for three-machine flow shop scheduling", *Oper. Res.* , to appear. *(SS15)*

Chen, B., Potts, C. N., and Strusevich, V. A. (1995), "Approximation algorithms for two-machine flow shop scheduling with batch setup times", Technical Report 152, Warwick Business School Research Paper, University of Warwick. *(SS16)*

Chen, B., and Strusevich, V. A. (1993a), "Approximation algorithms for three-machine open shop scheduling", *ORSA J. Comput.* **5**, 321–326. *(SS14)*

Chen, B., and Strusevich, V. A. (1993b), "Worst-case analysis of heuristics for open shops with parallel machines", *European J. Oper. Res.* **70**, 379–390. *(SS14)*

Cheriyan, J., and Thurimella, R. (1996), "Approximating minimum-size k-connected spanning subgraps via matching", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 292–301. *(ND24, ND25)*

Choi, J., Sellen, J., and Yap, C. K. (1994), "Approximate Euclidean shortest path motion planning", *Proc. 10th Ann. ACM Symp. Comput. Geom.*, ACM, . *(MS11)*

Chor, B., and Sudan, M. (1995), "A geometric approach to betweenness", *Proc. 3rd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 979, Springer-Verlag, 227–237. *(MS1)*

Christofides, N. (1976), "Worst-case analysis of a new heuristic for the travelling salesman problem", Technical report, Graduate School of Industrial Administration, Carnegie-Mellon University, Pittsburgh. *(ND30)*

Chudak, F. A., and Shmoys, D. B. (1997), "Approximation algorithms for precedence-constrained scheduling problems on parallel machines that run at different speed", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 581–590. *(SS7)*

Chvátal, V. (1979), "A greedy heuristic for the set covering problem", *Math. Oper. Res.* **4**, 233–235. *(SP4)*

Clementi, A., and Ianni, M. Di (1996), "On the hardness of approximating optimum schedule problems in store and forward networks", *IEEE/ACM Transaction on Networking* **4**, 272–280. *(SS19)*

Clementi, A., and Trevisan, L. (1996), "Improved non-approximability results for vertex cover problems with density constraints", *Proc. 2nd Ann. Int. Conf. on Computing and Combinatorics*, Lecture Notes in Comput. Sci. 1090, Springer-Verlag, 333–342. *(GT1)*

Coffman, E. G., Garey, M. R., Johnson, D. S., and Lapaugh, A. S. (1985), "Scheduling file transfers", *SIAM J. Comp.* **14**, 744–780. *(SS18)*

Coffman, E. G., Jr, Garey, M. R., and Johnson, D. S. (1984), "Approximation algorithms for bin-packing – an updated survey", in *Algorithm Design for Computer System Design*, Springer-Verlag, New York, 49–106. *(SR1)*

Cornuejols, G., Fisher, M., and Nemhauser, G. (1977), "Location of bank accounts to optimize float: An analytic study of exact and approximate algorithms", *Management Sci.* **23**, 789–810. *(ND55)*

Cowen, L. J., Goddard, W., and Jesurum, C. E. (1997), "Coloring with defect", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 548–557. *(GT5)*

Crama, Y., and Spieksma, F. C. R. (1992), "Approximation algorithms for three-dimensional assignment problems with triangle inequalities", *European J. Oper. Res.* **60**, 273–279. *(SP10)*

Crescenzi, P., Kann, V., Silvestri, R., and Trevisan, L. (1995), "Structure in approximation classes", *Proc. 1st Ann. Int. Conf. on Computing and Combinatorics*, Lecture Notes in Comput. Sci. 959, Springer-Verlag, 539–548. *(GT7, ND2, SR1)*

Crescenzi, P., and Panconesi, A. (1991), "Completeness in approximation classes", *Inform. and Comput.* **93**, 241–262. *(LO8)*

Crescenzi, P., Silvestri, R., and Trevisan, L. (1994), "On the query complexity of complete problems in approximation classes", Unpublished manuscript.

Crescenzi, P., and Trevisan, L. (1994), "On approximation scheme preserving reducibility and its applications", *Proc. 14th Ann. Conf. on Foundations of Software Tech. and Theoret. Comput. Sci.*, Lecture Notes in Comput. Sci. 880, Springer-Verlag, 330–341.

Dahlhaus, E., Johnson, D. S., Papadimitriou, C. H., Seymour, P. D., and Yannakakis, M. (1994), "The complexity of multiterminal cuts", *SIAM J. Comp.* **23**, 864–894. *(ND18)*

dAnzeo, C. (1996), "Optimization complexity of the scs problem given a longest common subsequence", Unpublished manuscript. *(SR4)*

DasGupta, B., He, X., Jiang, T., Li, M., Tromp, J., and Zhang, L. (1997), "On distances between phylogenetic trees", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 427–436. *(MS13)*

Datta, A. K., and Sen, R. K. (1995), "1-approximation algorithm for bottleneck disjoint path matching", *Inform. Process. Lett.* **55**, 41–44. *(GT12)*

Doddi, S., Marathe, M. V., Mirzaian, A., Moret, B. M. E., and Zhu, B. (1997), "Map labeling and its generalizations", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 148–157. *(MS12)*

Dudek, G., Romanik, K., and Whitesides, S. (1994), "Localizing a robot with minimum travel", Technical Report SOCS-94.5, McGill University. *(GP2)*

Eades, P., and Wormald, N. C. (1994), "Edge crossings in drawings of bipartite graphs", *Algorithmica* **11**, 379–403. *(ND12)*

Eppstein, D. (1992), "Approximating the minimum weight triangulation", *Proc. Third Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 48–57. *(ND58)*

Errico, B., and Rosati, R. (1995), "Minimal models in propositional logics: approximation results", *Proc. of 5th Italian Conference on Theoretical Computer Science*, Word Scientific, 547–562. *(LO7)*

Even, G., Naor, J., Rao, S., and Schieber, B. (1997), "Fast approximate graph partitioning algorithms", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 639–648. *(ND21)*

Even, G., Naor, J., Schieber, B., and Sudan, M. (1995), "Approximating minimum feedback sets and multi-cuts in directed graphs", *Proc. 4th Int. Conf. on Integer Prog. and Combinatorial Optimization*, Lecture Notes in Comput. Sci. 920, Springer-Verlag, 14–28. *(GT8, GT9)*

Even, G., Naor, J., and Zosin, L. (1996), "An 8-approximation algorithm for the subset feedback vertex set problem", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 310–319. *(GT8)*

Farach, M., Kannan, S., and Warnow, T. (1993), "A robust model for finding optimal evolutionary trees", *Proc. 25th Ann. ACM Symp. on Theory of Comp.*, ACM, 137–145. *(MS7)*

Feder, T., and Greene, D. H. (1988), "Optimal algorithms for approximate clustering", *Proc. 20th Ann. ACM Symp. on Theory of Comp.*, ACM, 434–444. *(ND33, ND48, ND49)*

Feige, U. (1996), "A threshold of $\ln n$ for approximating set cover", *Proc. 28th Ann. ACM Symp. on Theory of Comp.*, ACM, 314–318. *(GT2, SP4)*

Feige, U., and Goemans, M. X. (1995), "Approximating the value of two prover proof systems, with applications to MAX 2SAT and MAX DICUT", *Proc. 3rd Israel Symp. on Theory of Computing and Systems*, IEEE Computer Society, 182–189. *(ND13, LO2)*

Feige, U., and Kilian, J. (1996), "Zero knowledge and the chromatic number", *Proc. Comp. Complexity*, , . *(GT5)*

Feige, U., Kortsarz, G., and Peleg, D. (1995), "The dense k-subgraph problem", Unpublished manuscript. *(GT32)*

Fernandes, C. G. (1997), "A better approximation ratio for the minimum k-edge-connected spanning subgraph problem", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 629–638. *(ND25)*

Fernandez de la Vega, W., and Zissimopoulos, V. (1991), "An approximation scheme for strip-packing of rectangles with bounded dimensions", Technical Report 713, Laboratoire de Recherche en Informatique, Université de Paris, Orsay. *(SR2)*

Floréen, P., and Orponen, P. (1993), "Attraction radii in binary Hopfield nets are hard to compute", *Neural Comput.* **5**, 812–821. *(MS4)*

Formann, M., and Wagner, F. (1991), "A packing problem with applications to lettering of maps", *Proc. 7th Annual ACM Symposium on Computational Geometry*, ACM, 281–288. *(MS12)*

Frederickson, G. N. (1979), "Approximation algorithms for some postman problems", *J. ACM* **26**, 538–554. *(ND34)*

Frederickson, G. N., Hecht, M. S., and Kim, C. E. (1978), "Approximation algorithms for some routing problems", *SIAM J. Comp.* **7**, 178–193. *(ND32, ND35, ND36, ND37)*

Frederickson, G. N., and Jájá, J. (1981), "Approximation algorithms for several graph augmentation problems", *SIAM J. Comp.* **10**, 270–283. *(ND26, ND27)*

Frederickson, G. N., and Jájá, J. (1982), "On the relationship between the biconnectivity augmentation and traveling salesman problems", *Theoretical Computer Science* **19**, 189–201. *(ND24, ND26)*

Frieze, A., Galbiati, G., and Maffioli, F. (1982), "On the worst-case performance of some algorithms for the asymmetric traveling salesman problem", *Networks* **12**, 23–39. *(ND30)*

Frieze, A., and Jerrum, M. (1995), "Improved approximation algorithms for MAX k-CUT and MAX BISECTION", *Proc. 4th Int. Conf. on Integer Prog. and Combinatorial Optimization*, Lecture Notes in Comput. Sci. 920, Springer-Verlag, 1–13. *(GT31, ND11, ND14)*

Fürer, M., and Raghavachari, B. (1994), "Approximating the minimum-degree Steiner tree to within one of optimal", *J. Algorithms* **17**, 409–423. *(ND2)*

Galbiati, G., Maffioli, F., and Morzenti, A. (1994), "A short note on the approximability of the maximum leaves spanning tree problem", *Inform. Process. Lett.* **52**, 45–49. *(ND4)*

Galbiati, G., Maffioli, F., and Morzenti, A. (1995), "On the approximability of some maximum spanning tree problems", *Proc. 2nd Int. Symp. Latin American Theoretical Informatics*, Lecture Notes in Comput. Sci. 911, Springer-Verlag, 300–311. *(ND4)*

Garey, M., and Graham, R. (1975), "Bounds for multiprocessor scheduling with resource constraints", *SIAM J. Comp.* **4**, 187–200. *(SS8)*

Garey, M. R., and Johnson, D. S. (1979), *Computers and Intractability: a guide to the theory of NP-completeness*, W. H. Freeman and Company, San Francisco. *(ND2, SR1)*

Garg, A., and Tamassia, R. (1994), "On the computational complexity of upward and rectilinear planarity testing", *Proc. DIMACS Int. Workshop on Graph Drawing*, Lecture Notes in Comput. Sci. 894, Springer-Verlag, 286–297. *(ND57)*

Garg, N. (1996), "A 3-approximation for the minimum tree spanning k vertices", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 302–309. *(ND1)*

Garg, N., Santosh, V. S., and Singla, A. (1993), "Improved approximation algorithms for biconnected subgraphs via better lower bounding techniques", *Proc. 4th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 103–111. *(ND24)*

Garg, N., Saran, H., and Vazirani, V. (1994), "Finding separator cuts in planar graphs within twice the optimal", *Proc. 35th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 14–23. *(ND21)*

Garg, N., Vazirani, V. V., and Yannakakis, M. (1993), "Primal-dual approximation algorithms for integral flow and multicut in trees, with applications to matching and set cover", *Proc. 20th Int. Colloquium on Automata, Languages and Programming*, Lecture Notes in Comput. Sci. 700, Springer-Verlag, 64–75. *(ND19, ND43, SP4)*

Garg, N., Vazirani, V. V., and Yannakakis, M. (1994), "Multiway cuts in directed and node weighted graphs", *Proc. 21st Int. Colloquium on Automata, Languages and Programming*, Lecture Notes in Comput. Sci. 820, Springer-Verlag, 487–498. *(GT24, ND17, ND18, ND19)*

Garg, N., Vazirani, V. V., and Yannakakis, M. (1996), "Approximate max-flow min-(multi)cut theorems and their applications", *SIAM J. Comp.* **25**, 235–251. *(GT30, ND19, LO11)*

Gens, G. V., and Levner, E. V. (1979), "Computational complexity of approximation algorithms for combinatorial problems", *Proc. 8th International Symp. on Mathematical Foundations of Comput. Sci.*, Lecture Notes in Comput. Sci. 74, Springer-Verlag, 292–300. *(MP13, MP15)*

Gil, J., and Itai, A. (1995), "Packing trees", *Proc. 3rd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 979, Springer-Verlag, 113–127. *(SR10)*

Goemans, M. X. (1994), "An approximation algorithm for scheduling on three dedicated machines", *Disc. Appl. Math.* , to appear. *(SS13)*

Goemans, M. X. (1997), "Improved approximation algorithms for scheduling with release dates", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 591–598. *(SS4)*

Goemans, M. X., Goldberg, A. V., Plotkin, S., Shmoys, D. B., Tardos, É., and Williamson, D. P. (1994), "Improved approximation algorithms for network design problems", *Proc. 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 223–232. *(ND9)*

Goemans, M. X., and Williamson, D. P. (1995a), "A general approximation technique for constrained forest problems", *SIAM J. Comp.* **24**, 296–317. *(GT14, GT48, ND7, ND30)*

Goemans, M. X., and Williamson, D. P. (1995b), "Improved approximation algorithms for maximum cut and satisfiability problems using semidefinite programming", *J. ACM* **42**, 1115–1145. *(ND11, LO1)*

Goemans, M. X., and Williamson, D. P. (1996), "Primal-dual approximation algorithms for feedback problems in planar graphs", *Proc. 5th Int. Conf. on Integer Prog. and Combinatorial Optimization*, Lecture Notes in Comput. Sci. 1084, Springer-Verlag, 147–161. *(GT8, GT9, GT30)*

Goldberg, L. A., Paterson, M., Srinivasan, A., and Sweedyk, E. (1997), "Better approximation guarantees for job-shop scheduling", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 599–608. *(SS17)*

Goldschmidt, O., and Hochbaum, D. S. (1988), "Polynomial algorithm for the k-cut problem", *Proc. 29th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 444–451. *(ND16)*

Gonzalez, T. F. (1985), "Clustering to minimize the maximum intercluster distance", *Theoretical Computer Science* **38**, 293–306. *(ND49)*

Gonzalez, T. F., and Zheng, S. (1989), "Improved bounds for rectangular and Guilhotine partitions", *J. Symbolic Comput.* **7**, 591–610. *(MS8)*

Gonzalez, T. F., and Zheng, S. (1990), "Approximation algorithm for partitioning a rectangle with interior points", *Algorithmica* **5**, 11–42. *(MS8)*

Grigoriadis, M. D., and Khachiyan, L. G. (1994), "Fast approximation schemes for convex programs with many blocks and coupling constraints", *SIAM J. Optimization* **4**, 86–107. *(MP17)*

Guha, S., and Khuller, S. (1996), "Approximation algorithms for connected dominating sets", *Proc. 4th Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 1136, Springer-Verlag, 179–193. *(GT2)*

Gusfield, D., and Pitt, L. (1992), "A bounded approximation for the minimum cost 2-sat problem", *Algorithmica* **8**, 103–117. *(LO7)*

Guttman, N., and Hassin, R. (1996), "Approximation algorithms for minimum sum p-checking", Unpublished manuscript. *(ND50)*

Hall, L. A., Schulz, A. S., Shmoys, D. B., and Wein, J. (1997), "On-line and off-line approximation algorithms", Unpublished manuscript. *(SS4)*

Hall, N. G., and Hochbaum, D. S. (1986), "A fast approximation algorithm for the multicovering problem", *Disc. Appl. Math.* **15**, 35–40. *(MP1)*

Halldórsson, M. M. (1993a), "A still better performance guarantee for approximate graph coloring", *Inform. Process. Lett.* **45**, 19–23. *(GT5, GT13)*

Halldórsson, M. M. (1993b), "Approximating the minimum maximal independence number", *Inform. Process. Lett.* **46**, 169–172. *(GT4)*

Halldórsson, M. M. (1994), "personal communication", Unpublished manuscript. *(GT15, GT16, GT34, GT42, SP1)*

Halldórsson, M. M. (1995a), "Approximating discrete collections via local improvements", *Proc. 6th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 160–169. *(GT1, GT21, GT34)*

Halldórsson, M. M. (1995b), "Approximation via partitioning", Technical Report IS-RR-95-0003F, School of Information Science, Japan Advanced Institute of Science and Technology, Hokuriku. *(GT20, GT21, GT22, GT23, SR6, MP10)*

Halldórsson, M. M. (1996), "Approximating k-set cover and complementary graph coloring", *Proc. 5th Int. Conf. on Integer Prog. and Combinatorial Optimization*, Lecture Notes in Comput. Sci. 1084, Springer-Verlag, 118–131. *(GT5, GT13, GT15, SP4)*

Halldórsson, M. M., Iwano, K., Katoh, N., and Tokuyama, T. (1995), "Finding subsets maximizing minimum structures", *Proc. 6th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 150–159. *(ND5)*

Halldórsson, M. M., and Radhakrishnan, J. (1994), "Improved approximations of independent sets in bounded-degree graphs", *Nordic J. Comp.* **1**, 475–492. *(GT21)*

Halldórsson, M. M., and Tanaka, K. (1996), "Approximation and special cases of common subtrees and editing distance", *Proc. 7th Ann. Int. Symp. on Algorithms and Computation*, Lecture Notes in Comput. Sci. 1178, , 75–84. *(GT44)*

Halldórsson, M. M., Ueno, S., Nakao, H., and Kajitani, Y. (1992), "Approximating Steiner trees in graphs with restricted weights", *Proc. Asia-Pacific Conference on Circuits and Systems, Sidney, Australia*, , 69–73. *(ND7)*

Haralambides, J., Makedon, F., and Monien, B. (1991), "Bandwidth minimization: an approximation algorithm for caterpillars", *Math. Systems Theory* **24**, 169–177. *(GT39)*

Hassin, R. (1992), "Approximation schemes for the restricted shortest path problem", *Math. Oper. Res.* **17**, 36–42. *(ND40)*

Hassin, R., and Megiddo, N. (1991), "Approximation algorithms for hitting objects with straight lines", *Disc. Appl. Math.* **30**, 29–42. *(SP7)*

Hassin, R., and Rubinstein, S. (1994), "Approximations for the maximum acyclic subgraph problem", *Inform. Process. Lett.* **51**, 133–140. *(GT9)*

Hassin, R., Rubinstein, S., and Tamir, A. (1994), "Notes on dispersion problems", Unpublished manuscript. *(GT32)*

Håstad, J. (1996), "Clique is hard to approximate within $n^{1-\varepsilon}$", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 627–636. *(GT20)*

Håstad, J. (1997), "Some optimal inapproximability results", *Proc. 29th Ann. ACM Symp. on Theory of Comp.*, ACM, to appear. *(GT1, GT30, ND11, ND13, MP10, LO2)*

Håstad, J., Phillips, S., and Safra, S. (1993), "A well-characterized approximation problem", *Inform. Process. Lett.* **47**, 301–305. *(AN1)*

Hochbaum, D. S. (1982), "Approximation algorithms for the set covering and vertex cover problems", *SIAM J. Comp.* **11**, 555–556. *(GT1, SP4)*

Hochbaum, D. S. (1983), "Efficient bounds for the stable set, vertex cover and set packing problems", *Disc. Appl. Math.* **6**, 243–254. *(GT21, SP2)*

Hochbaum, D. S. (1997), "Various notions of approximations: good, better, best, and more", in *Approximation algorithms for NP-hard problems*, PWS Publishing Company, Boston, 346–398. *(ND48)*

Hochbaum, D. S., and Maass, W. (1985), "Approximation schemes for covering and packing problems in image processing and VLSI", *J. ACM* **32**, 130–136. *(SP8)*

Hochbaum, D. S., and Maass, W. (1987), "Fast approximation algorithms for a nonconvex covering problem", *J. Algorithms* **8**, 305–323. *(SP8)*

Hochbaum, D. S., Megiddo, N., Naor, J., and Tamir, A. (1993), "Tight bounds and 2-approximation algorithms for integer programs with two variables per inequality", *Math. Programming* **62**, 69–83. *(MP1)*

Hochbaum, D. S., and Shmoys, D. B. (1986), "A unified approach to approximation algorithms for bottleneck problems", *J. ACM* **33**, 533–550. *(ND33, ND48, ND49, ND51, ND56)*

Hochbaum, D. S., and Shmoys, D. B. (1987), "Using dual approximation algorithms for scheduling problems: theoretical and practical results", *J. ACM* **34**, 144–162. *(SS6)*

Hochbaum, D. S., and Shmoys, D. B. (1988), "A polynomial approximation scheme for machine scheduling on uniform processors: using the dual approach", *SIAM J. Comp.* **17**, 539–551. *(SS10)*

Holyer, I. (1981), "The NP-completeness of edge-coloring", *SIAM J. Comp.* **10**, 718–720. *(GT7)*

Hoogeveen, J. A., Lenstra, J. K., and Veltman, B. (1995), "Three, four, five, six, or the complexity of scheduling with communication delays", *Oper. Res. Lett.* **to appear**, . *(SS7)*

Horowitz, E., and Sahni, S. (1978), *Fundamentals of computer algorithms*, Pitman, . *(GT5)*

Horowitz, E., and Sahni, S. K. (1976), "Exact and approximate algorithms for scheduling nonidentical processors", *J. ACM* **23**, 317–327. *(SS6, SS10)*

Hsu, W. L., and Nemhauser, G. L. (1979), "Easy and hard bottleneck location problems", *Disc. Appl. Math.* **1**, 209–216. *(ND48)*

Hunt III, H. B., Marathe, M. V., Radhakrishnan, V., Ravi, S. S., Rosenkrantz, D. J., and Stearns, R. E. (1994a), "Approximation schemes using L-reductions", *Proc. 14th Ann. Conf. on Foundations of Software Tech. and Theoret. Comput. Sci.*, Lecture Notes in Comput. Sci. 880, Springer-Verlag, 342–353. *(GT34)*

Hunt III, H. B., Marathe, M. V., Radhakrishnan, V., Ravi, S. S., Rosenkrantz, D. J., and Stearns, R. E. (1994b), "A unified approach to approximation schemes for NP- and PSPACE-hard problems for geometric graphs", *Proc. 2nd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 855, Springer-Verlag, 424–435. *(GT1, GT2, GT3, GT10, GT11, GT21)*

Hurkens, C. A. J., and Schrijver, A. (1989), "On the size of systems of sets every t of which have an SDR, with an application to the worst-case ratio of heuristics for packing problems", *SIAM J. Disc. Math.* **2**, 68–72. *(GT10, GT11, SP1, SP2)*

Ianni, M. Di (1996), "Efficient delay routing", *2nd International EURO-PAR Conference*, , Lecture Notes in Comput. Sci., Springer-Verlag, . *(SS19)*

Ibarra, O. H., and Kim, C. E. (1975), "Fast approximation for the knapsack and sum of subset problems", *J. ACM* **22**, 463–468. *(MP13)*

Ihler, E. (1991), "Bounds on the quality of approximate solutions to the group Steiner problem", *Proc. 17th Workshop on Graph-Theoretic Concepts in Computer Science*, Lecture Notes in Comput. Sci. 484, Springer-Verlag, 109–118. *(ND7, ND8)*

Ihler, E. (1992), "The complexity of approximating the class Steiner tree problem", *Proc. 18th Workshop on Graph-Theoretic Concepts in Computer Science*, Lecture Notes in Comput. Sci. 570, Springer-Verlag, 85–96. *(ND7)*

Jagota, A. (1993), "Constraint satisfaction and maximum clique", *Working Notes, AAAI Spring Symposium on AI and NP-hard Problems*, Stanford University, 92–97. *(MS10)*

Jansen, K. (1992), "An approximation algorithm for the general routing problem", *Inform. Process. Lett.* **41**, 333–339. *(ND38)*

Jansen, K., and Öhring, S. (1997), “Approximation algorithms for time constrained scheduling”, *Inform. and Comput.* **132**, 85–108. *(SR1)*

Jiang, T., Lawler, E. L., and Wang, L. (1994), “Aligning sequences via an evolutionary tree: complexity and approximation”, *Proc. 26th Ann. ACM Symp. on Theory of Comp.*, ACM, 760–769. *(ND7, MS3)*

Jiang, T., and Li, M. (1994a), “Approximating shortest superstrings with constraints”, *Theoretical Computer Science* **134**, 473–491. *(SR5)*

Jiang, T., and Li, M. (1994b), “On the approximation of shortest common supersequences and longest common subsequences”, *Proc. 21st Int. Colloquium on Automata, Languages and Programming*, Lecture Notes in Comput. Sci. 820, Springer-Verlag, 191–202. *(SR4, SR6)*

Jiang, T., and Wang, L. (1994), “An approximation scheme for some Steiner tree problems in the plane”, *Proc. 5th Ann. Int. Symp. on Algorithms and Computation*, Lecture Notes in Comput. Sci. 834, Springer-Verlag, 414–422. *(ND8)*

Johnson, D. S. (1974), “Approximation algorithms for combinatorial problems”, *J. Comput. System Sci.* **9**, 256–278. *(GT2, SP4, SP5, LO2)*

Johnson, D. S. (1990), “A catalog of complexity classes”, in *Algorithms and Complexity*, volume A of *Handbook of Theoretical Computer Science*, Elsevier science publishing company, Amsterdam, 67–161.

Johnson, D. S., and Garey, M. R. (1985), “A 71/60 theorem for bin-packing”, *J. Complexity* **1**, 65–106. *(SR1)*

Jonsson, P. (1997), “Tight lower bounds on the approximability of some NPO PB-complete problems”, Technical Report 4, Department of Computer and Information Science, Linköping University, Sweden. *(MP2, LO6, LO7)*

Kann, V. (1991), “Maximum bounded 3-dimensional matching is MAX SNP-complete”, *Inform. Process. Lett.* **37**, 27–35. *(GT10, SP1, SP2)*

Kann, V. (1992a), “On the approximability of the maximum common subgraph problem”, *Proc. 9th Ann. Symp. on Theoretical Aspects of Comput. Sci.*, Lecture Notes in Comput. Sci. 577, Springer-Verlag, 377–388. *(GT42, GT43)*

Kann, V. (1992b), *On the Approximability of NP-complete Optimization Problems*, PhD thesis, Department of Numerical Analysis and Computing Science, Royal Institute of Technology, Stockholm. *(GT2, GT4, GT8, GT9, GT42, SP6, LO6, LO9)*

Kann, V. (1994a), “Maximum bounded H-matching is MAX SNP-complete”, *Inform. Process. Lett.* **49**, 309–318. *(GT11)*

Kann, V. (1994b), “Polynomially bounded minimization problems that are hard to approximate”, *Nordic J. Comp.* **1**, 317–331. *(GT4, GT47, MP1, LO7, LO10, AL3)*

Kann, V. (1995), “Strong lower bounds on the approximability of some NPO PB-complete maximization problems”, *Proc. 20th International Symp. on Mathematical Foundations of Comput. Sci.*, Lecture Notes in Comput. Sci. 969, Springer-Verlag, 227–236. *(GT26, MP2, MP9)*

Kann, V., Khanna, S., Lagergren, J., and Panconesi, A. (1997), “Hardness of approximating MAX k-CUT and its dual”, *Chicago Journal of Theoretical Computer Science* , . *(GT30, ND14)*

Kann, V., Lagergren, J., and Panconesi, A. (1996), “Approximability of maximum splitting of k-sets and some other APX-complete problems”, *Inform. Process. Lett.* **58**, 105–110. *(SP3, LO4)*

Karger, D., Motwani, R., and Ramkumar, G. D. S. (1993), “On approximating the longest path in a graph”, *Proc. 3rd Workshop on Algorithms and Data Structures*, Lecture Notes in Comput. Sci. 709, Springer-Verlag, 421–432. *(ND39)*

Karger, D., Motwani, R., and Sudan, M. (1994), "Approximate graph coloring by semidefinite programming", *Proc. 35th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 2–13. *(GT5)*

Karmarkar, N., and Karp, R. M. (1982), "An efficient approximation scheme for the one-dimensional bin packing problem", *Proc. 23rd Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 312–320. *(SR1)*

Karp, R. M., McKellar, A. C., and Wong, C. K. (1975), "Near-optimal solutions to a 2-dimensional placement problem", *SIAM J. Comp.* **4**, 271–286. *(MP8)*

Karpinski, M., and Zelikovsky, A. (1995), "New approximation algorithms for the Steiner tree problems", Technical Report TR95-030, Electronic Colloquium on Computational Complexity. *(ND7, ND8)*

Karuno, Y., Nagamochi, H., and Ibaraki, T. (1993), "Vehicle scheduling on a tree with release and handling times", *Proc. 4th Ann. Int. Symp. on Algorithms and Computation*, Lecture Notes in Comput. Sci. 762, Springer-Verlag, 486–495. *(SS20)*

Kavvadias, D., Papadimitriou, C. H., and Sideri, M. (1993), "On Horn envelopes and hypergraph transversals", *Proc. 4th Ann. Int. Symp. on Algorithms and Computation*, Lecture Notes in Comput. Sci. 762, Springer-Verlag, 399–405. *(LO13)*

Kellerer, H., Tautenhahn, T., and Woeginger, G.J. (1996), "Approximability and nonapproximability results for minimizing total flow time on a single machine", *Proc. 28th Ann. ACM Symp. on Theory of Comp.*, ACM, 418–426. *(SS11)*

Kenyon, C., and Rémila, E. (1996), "Approximate strip packing", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 31–36. *(SR2)*

Khanna, S., Linial, N., and Safra, S. (1993), "On the hardness of approximating the chromatic number", *Proc. 2nd Israel Symp. on Theory of Computing and Systems*, IEEE Computer Society, 250–260. *(GT5)*

Khanna, S., and Motwani, R. (1996), "Toward a syntactic characterization of PTAS", *Proc. 28th Ann. ACM Symp. on Theory of Comp.*, ACM, 329–337. *(LO1)*

Khanna, S., Motwani, R., Sudan, M., and Vazirani, U. (1994), "On syntactic versus computational views of approximability", *Proc. 35th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 819–830. *(GT2, SP4)*

Khanna, S., Motwani, R., and Yao, F. F. (1995), "Approximation algorithms for the largest common subtree problem", Unpublished manuscript. *(SR7)*

Khuller, S. (1997), "Approximation algorithms for finding highly connected subgraphs", in *Approximation algorithms for NP-hard problems*, PWS Publishing Company, Boston, 236–265. *(ND26)*

Khuller, S., Pless, R., and Sussmann, Y. J. (1997), "Fault tolerant k-center problems", *Proc. 3rd Italian Conf. on Algorithms and Complexity*, Lecture Notes in Comput. Sci. 1203, Springer-Verlag, 37–48. *(ND48)*

Khuller, S., and Raghavachari, B. (1995), "Improved approximation algorithms for uniform connectivity problems", *Proc. 27th Ann. ACM Symp. on Theory of Comp.*, ACM, 1–10. *(ND24, ND25)*

Khuller, S., Raghavachari, B., and Rosenfeld, A. (1994), "Localization in graphs", Technical Report UMIACS-TR-94-92, University of Maryland, UMIACS. *(GT49)*

Khuller, S., Raghavachari, B., and Young, N. (1993), "Maintaining directed reachability with few edges", Technical Report UMIACS-TR-93-87, University of Maryland, UMIACS. *(ND10)*

Khuller, S., Raghavachari, B., and Young, N. (1995), "Approximating the minimum equivalent digraph", *SIAM J. Comp.* **24**, 859–872. *(GT35)*

Khuller, S., Raghavachari, B., and Young, N. (1996a), "Low degree spanning trees of small weight", *SIAM J. Comp.* **25**, 355–368. *(ND3)*

Khuller, S., Raghavachari, B., and Young, N. (1996b), "On strongly connected digraphs with bounded cycle length", *Disc. Appl. Math.* **69**, 281–289. *(ND24, ND25, ND27)*

Khuller, S., and Sussmann, Y. J. (1996), "The capacitated k-center problem", *Proc. 4th Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 1136, Springer-Verlag, 152–166. *(ND48)*

Khuller, S., and Thurimella, R. (1993), "Approximation algorithms for graph augmentation", *J. Algorithms* **14**, 214–225. *(ND26)*

Khuller, S., and Vishkin, U. (1994), "Biconnectivity approximations and graph carvings", *J. ACM* **41**, 214–235. *(ND9, ND25)*

Kierstead, H.A. (1991), "A polynomial time approximation algorithm for dynamic storage allocation", *Disc. Math.* **88**, 231–237. *(SR3)*

Kim, S., and McNaughton, R. (1993), "Computing the order of a locally testable automaton", Unpublished manuscript. *(AL4)*

Klein, P., Agrawal, A., Ravi, R., and Rao, S. (1990), "Approximation through multicommodity flow", *Proc. 31st Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 726–737. *(GT37, PO1)*

Klein, P., Plotkin, S. A., and Rao, S. (1993), "Excluded minors, network decomposition, and multicommodity flow", *Proc. 25th Ann. ACM Symp. on Theory of Comp.*, ACM, 682–690. *(ND20)*

Kleinberg, J., and Tardos, É. (1995), "Approximations for the disjoint paths problem in high-diameter planar networks", *Proc. 27th Ann. ACM Symp. on Theory of Comp.*, ACM, 26–35. *(ND44)*

Kloks, T., Kratsch, D., and Müller, H. (1995), "Approximating the bandwidth for asteroidal triple-free graphs", *Proc. 3rd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 979, Springer-Verlag, 434–447. *(GT39)*

Ko, M. T., Lee, R. C. T., and Chang, J. S. (1990), "An optimal approximation algorithm for the rectilinear m-center problem", *Algorithmica* **5**, 341–352. *(ND48)*

Kohli, R., Krishnamurti, R., and Mirchandani, P. (1994), "The minimum satisfiability problem", *SIAM J. Disc. Math.* **7**, 275–283. *(LO2, LO3)*

Kolaitis, P. G., and Thakur, M. N. (1994), "Logical definability of NP optimization problems", *Inform. and Comput.* **115**, 321–353. *(LO5)*

Kolaitis, P. G., and Thakur, M. N. (1995), "Approximation properties of NP minimization classes", *J. Comput. System Sci.* **50**, 391–411. *(GT24, GT25)*

Kortsarz, G., and Peleg, D. (1992a), "Approximation algorithms for minimum time broadcast", *Proc. 1st Israel Symp. on Theory of Computing and Systems*, Lecture Notes in Comput. Sci. 601, Springer-Verlag, 67–78. *(ND47)*

Kortsarz, G., and Peleg, D. (1992b), "Generating sparse 2-spanners", *Proc. 3rd Scandinavian Workshop on Algorithm Theory*, Lecture Notes in Comput. Sci. 621, Springer-Verlag, 73–82. *(GT33)*

Kortsarz, G., and Peleg, D. (1993), "On choosing a dense subgraph", *Proc. 34th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 692–701. *(GT32)*

Kortsarz, G., and Peleg, D. (1994), "Generating low-degree 2-spanners", *Proc. 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 556–563. *(GT33)*

Kortsarz, G., and Peleg, D. (1997), "Approximating shallow-light trees", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 103–110. *(ND7)*

Kosaraju, S. R., Park, J. K., and Stein, C. (1994), "Long tours and short superstrings", *Proc. 35th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 166–177. *(ND29)*

Kou, L. T., Stockmeyer, L. J., and Wong, C. K. (1978), "Covering edges by cliques with regard to keyword conflicts and intersection graphs", *Communications of the ACM* **21**, 135–139. *(GT15)*

Lam, S., and Sethi, R. (1977), "Worst case analysis of two scheduling algorithms", *SIAM J. Comp.* **6**, 518–536. *(SS7)*

Leighton, T., and Rao, S. (1988), "An approximate max-flow min-cut theorem for uniform multicommodity flow problems with applications to approximation algorithms", *Proc. 29th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 422–431. *(GT40, GT41, ND23)*

Lenstra, J. K., and Kan, A. H. G. Rinnooy (1978), "Complexity of scheduling under precedence constraints", *Oper. Res.* **26**, 22–35. *(SS7)*

Lenstra, J. K., and Shmoys, D. B. (1995), "Computing near-optimal schedules", in *Scheduling theory and its applications*, Wiley, Chichester, to appear. *(SS14)*

Lenstra, J. K., Shmoys, D. B., and Tardos, É. (1990), "Approximation algorithms for scheduling unrelated parallel machines", *Math. Programming* **46**, 259–271. *(SS6)*

Leonardi, S., and Raz, D. (1997), "Approximating total flow time on parallel machines", *Proc. 29th Ann. ACM Symp. on Theory of Comp.*, ACM, to appear. *(SS11)*

Levcopoulos, C., and Gudmundsson, J. (1996), "Approximation algorithms for covering polygons with squares and similar problems", Technical Report LU-CS-TR:96-181, Department of Computer Science, Lund University, Sweden. *(SR9)*

Li, C., McCormick, S. T., and Simchi-Levi, D. (1990), "The complexity of finding two disjoint paths with min-max objective function", *Disc. Appl. Math.* **26**, 105–115. *(ND45)*

Li, C., McCormick, S. T., and Simchi-Levi, D. (1992), "On the minimum-cardinality-bounded-diameter and the bounded-cardinality-minimum-diameter edge addition problems", *Oper. Res. Lett.* **11**, 303–308. *(ND28)*

Li, K., and Cheng, K. (1990), "On three-dimensional packing", *SIAM J. Comp.* **19**, 847–867. *(SR2)*

Li, M. (1990), "Towards a DNA sequencing theory", *Proc. 31st Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 125–134. *(SR5)*

Lin, C. (1994), "Hardness of approximating graph transformation problem", *Proc. 5th Ann. Int. Symp. on Algorithms and Computation*, Lecture Notes in Comput. Sci. 834, Springer-Verlag, 74–82. *(GT45)*

Lin, J-H., and Vitter, J. S. (1992), "ε-approximations with minimum packing constraint violation", *Proc. 24th Ann. ACM Symp. on Theory of Comp.*, ACM, 771–782. *(ND52)*

Lipton, R. J., and Tarjan, R. E. (1979), "A separator theorem for planar graphs", *SIAM J. Appl. Math.* **36**, 177–189. *(ND22)*

Lu, H., and Ravi, R. (1992), "The power of local optimization: Approximation algorithms for maximum-leaf spanning tree", *Proc. Allerton Conf.*, , 533–542. *(ND4)*

Ludwig, W., and Tiwari, P. (1994), "Scheduling malleable and nonmalleable parallel tasks", *Proc. 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 167–176. *(SS8)*

Lund, C., and Yannakakis, M. (1993), "The approximation of maximum subgraph problems", *Proc. 20th Int. Colloquium on Automata, Languages and Programming*, Lecture Notes in Comput. Sci. 700, Springer-Verlag, 40–51. *(GT23, GT24, GT26)*

Lund, C., and Yannakakis, M. (1994), "On the hardness of approximating minimization problems", *J. ACM* **41**, 960–981. *(GT2, GT5, GT13, GT15, GT16, SP4, SP5, SR1)*

Mahajan, S., and J. Ramesh, 1995 (1995), "Derandomizing semidefinite programming based approximation algorithms", *Proc. 36th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 162–169. *(GT5, GT20, GT31, ND11, ND14)*

Makedon, F., and Tragoudas, S. (1990), "Approximating the minimum net expansion: near optimal solutions to circuit partitioning problems", *Proc. 16th Workshop on Graph-Theoretic Concepts in Computer Science*, Lecture Notes in Comput. Sci. 484, Springer-Verlag, 140–153. *(ND23)*

Malesinska, Ewa, and Panconesi, Alessandro (1996), "On the hardness of frequency allocation for hybrid networks", *Proc. 22nd Workshop on Graph-Theoretic Concepts in Computer Science*, Lecture Notes in Comput. Sci. 1197, , 308–322. *(MS14)*

Marathe, M. V., Breu, H., Hunt III, H. B., Ravi, S. S., and Rosenkrantz, D. J. (1994), "Simple heuristics for unit disk graphs", *Networks* , to appear. *(GT4, GT5)*

Marathe, M. V., Ravi, R., Sundaram, R., Ravi, S. S., Rosenkrantz, D. J., and Hunt III, H. B. (1995), "Bicriteria network design problems", *Proc. 22nd Int. Colloquium on Automata, Languages and Programming*, Lecture Notes in Comput. Sci. 944, Springer-Verlag, 487–498. *(ND47)*

Maruyama, O., and Miyano, S. (1995), "Graph inference from a walk for trees of bounded degree 3 is NP-complete", *Proc. 20th International Symp. on Mathematical Foundations of Comput. Sci.*, Lecture Notes in Comput. Sci. 969, Springer-Verlag, 257–266. *(GT51)*

Michel, C., Schroeter, H., and Srivastav, A. (1995), "Tsp and matching in printed circuit board assembly", *European Symposium on Operations Research*, , . *(ND30)*

Middendorf, M. (1994), "On the approximation of finding various minimal, maximal, and consistent sequences", *Proc. 5th Ann. Int. Symp. on Algorithms and Computation*, Lecture Notes in Comput. Sci. 834, Springer-Verlag, 306–314. *(SR4, SR6)*

Mitchell, J. S. B., Piatko, C., and Arkin, E. M. (1992), "Computing a shortest k-link path in a polygon", *Proc. 33rd Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 573–582. *(MS6)*

Mitchell, J. S. B., and Suri, S. (1992), "Separation and approximation of polyhedral objects", *Proc. Third Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 296–306. *(ND59)*

Monien, B., and Speckenmeyer, E. (1985), "Ramsey numbers and an approximation algorithm for the vertex cover problem", *Acta Informatica* **22**, 115–123. *(GT1)*

Motwani, R., and Naor, J. S. (1994), "On exact and approximate cut covers of graphs", Technical Report STAN-CS-TN-94-11, Department of Computer Science, Stanford University. *(GT19)*

Nishizeki, T., Asano, T., and Watanabe, T. (1983), "An approximation algorithm for the Hamiltonian walk problem on maximal planar graphs", *Disc. Appl. Math.* **5**, 211–222. *(ND30)*

Nishizeki, T., and Chiba, N. (1988), *Planar Graphs: Theory and Algorithms*, volume 32 of *Annals of Disc. Math.*, Elsevier science publishing company, Amsterdam. *(GT23, SP1)*

Nishizeki, T., and Kashiwagi, K. (1990), "On the 1.1 edge-coloring of multigraphs", *SIAM J. Disc. Math.* **3**, 391–410. *(GT7)*

Orponen, P., and Mannila, H. (1987), "On approximation preserving reductions: Complete problems and robust measures", Technical Report C-1987-28, Department of Computer Science, University of Helsinki. *(ND29, MP1, LO7)*

Panconesi, A., and Ranjan, D. (1993), "Quantifiers and approximation", *Theoretical Computer Science* **107**, 145–163. *(GT34, LO6)*

Papadimitriou, C. H. (1985), "An algorithm for shortest-path motion in three dimensions", *Inform. Process. Lett.* **20**, 259–263. *(MS11)*

Papadimitriou, C. H., Raghavan, P., Sudan, M., and Tamaki, H. (1994), "Motion planning on a graph", *Proc. 35th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 511–520. *(GP1)*

Papadimitriou, C. H., and Yannakakis, M. (1991), "Optimization, approximation, and complexity classes", *J. Comput. System Sci.* **43**, 425–440. *(GT1, GT2, GT9, GT21, GT31, ND11, ND13, SP4, LO1, LO2, LO4)*

Papadimitriou, C. H., and Yannakakis, M. (1993), "The traveling salesman problem with distances one and two", *Math. Oper. Res.* **18**, 1–11. *(ND30)*

Park, J. K., and Phillips, C. A. (1993), "Finding minimum-quotient cuts in planar graphs", *Proc. 25th Ann. ACM Symp. on Theory of Comp.*, ACM, 766–775. *(ND23)*

Paz, A., and Moran, S. (1981), "Non deterministic polynomial optimization problems and their approximations", *Theoretical Computer Science* **15**, 251–277. *(GT13)*

Peleg, D., Schechtman, G., and Wool, A. (1993), "Approximating bounded 0-1 integer linear programs", *Proc. 2nd Israel Symp. on Theory of Computing and Systems*, IEEE Computer Society, 69–77. *(SP4)*

Petrank, E. (1992), "The hardness of approximation: gap location", Technical Report 754, Computer Science Department, Technion, Israel Institute of Technology, Haifa, Israel. *(MS2)*

Petrank, E. (1994), "The hardness of approximation: gap location", *Computational Complexity* **4**, 133–157. *(GT1, GT7, SP3)*

Phillips, C., Stein, C., and Wein, J. (1995), "Scheduling jobs that arrive over time", *Proc. 4th Workshop on Algorithms and Data Structures*, Lecture Notes in Comput. Sci. 955, Springer-Verlag, 86–97. *(SS12)*

Phillips, C. A. (1993), "The network inhibition problem", *Proc. 25th Ann. ACM Symp. on Theory of Comp.*, ACM, 776–785. *(ND15, ND40)*

Pitt, L., and Warmuth, M. K. (1993), "The minimum consistent DFA problem cannot be approximated within any polynomial", *J. ACM* **40**, 95–142. *(AL1)*

Plaisted, D. A., and Hong, J. (1987), "A heuristic triangulation algorithm", *J. Algorithms* **8**, 405–437. *(ND58)*

Plesník, J. (1980), "On the computational complexity of centers locating in a graph", *Aplikace Matematiky* **25**, 445–452. *(ND48)*

Plesník, J. (1981), "The complexity of designing a network with minimum diameter", *Networks* **11**, 77–85. *(ND6)*

Plesník, J. (1982), "Complexity of decomposing graphs into factors with given diameters or radii", *Math. Slovaca* **32**, 379–388. *(ND53)*

Plesník, J. (1987), "A heuristic for the p-center problem in graphs", *Disc. Appl. Math.* **17**, 263–268. *(ND48)*

Plesník, J. (1988), "Two heuristics for the absolute p-center problem in graphs", *Math. Slovaca* **38**, 227–233. *(ND48)*

Provan, J. S. (1988), "An approximation scheme for finding Steiner trees with obstacles", *SIAM J. Comp.* **17**, 920–934. *(ND8)*

Queyranne, M. (1985), "Bounds for assembly line balancing heuristics", *Oper. Res.* **33**, 1353–1359. *(SR1)*

Queyranne, M. (1986), "Performance ratio of polynomial heuristics for triangle inequality quadratic assignment problems", *Oper. Res. Lett.* **4**, 231–234. *(MP7)*

Rabani, Y. (1996), "Path coloring on the mesh", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 400–409. *(ND44)*

Raghavan, P., and Thompson, C. D. (1991), "Multiterminal global routing: A deterministic approximation scheme", *Algorithmica* **6**, 73–82. *(ND41)*

Raghavan, P., and Upfal, E. (1994), "Efficient routing in all-optical networks", *Proc. 26th Ann. ACM Symp. on Theory of Comp.*, ACM, 134–143. *(ND44)*

Ravi, R. (1994a), "A primal-dual approximation algorithm for the Steiner forest problem", *Inform. Process. Lett.* **50**, 185–190. *(ND7)*

Ravi, R. (1994b), "Rapid rumor ramification: approximating the minimum broadcast time", *Proc. 35th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 202–213. *(ND47)*

Ravi, R., Agrawal, A., and Klein, P. (1991), "Ordering problems approximated: single-processor scheduling and interval graph completion", *Automata, Languages and Programming*, Lecture Notes in Comput. Sci. 510, Springer-Verlag, 751–762. *(GT36, SS2)*

Ravi, R., Sundaram, R., Marathe, M. V., Rosenkrantz, D. J., and Ravi, S. S. (1994), "Spanning trees short or small", *Proc. 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 546–555. *(ND1, ND7)*

Ravi, R., and Williamson, D. (1995), "An approximation algorithm for minimum-cost vertex-connectivity problems", *Proc. 6th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 332–341. *(ND24)*

Ravi, S. S., Rosenkrantz, D. J., and Tayi, G. K. (1991), "Facility dispersion problems: heuristics and special cases", *Proc. 2nd Workshop on Algorithms and Data Structures*, Lecture Notes in Comput. Sci. 519, Springer-Verlag, 355–366. *(ND54)*

Rayward-Smith, V. J. (1987), "Net scheduling with unit interprocessor communication delays", *Disc. Appl. Math.* **18**, 55–71. *(SS7)*

Sahni, S. K., and Gonzalez, T. F. (1976), "P-complete approximation problems", *J. ACM* **23**, 555–565. *(GT17, GT18, ND50, MP6, MP7)*

Salman, F. S., Cheriyan, J., Ravi, R., and Subramanian, S. (1997), "Buy-at-bulk network design: approximating the single-sink edge installation problem", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 619–628. *(ND46)*

Saran, H., and Vazirani, V. (1991), "Finding k-cuts within twice the optimal", *Proc. 32nd Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 743–751. *(ND16)*

Schiermeyer, I. (1994), "Reverse-fit: a 2-optimal algorithm for packing rectangles", *Proc. 2nd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 855, Springer-Verlag, 290–299. *(SR2)*

Seymour, P. D. (1995), "Packing directed circuits fractionally", *Combinatorica* **15**, 281–288. *(GT8)*

Seymour, P. D., and Thomas, R. (1994), "Call routing and the ratcatcher", *Combinatorica* **14**, 217–241. *(ND10)*

Shmoys, D. B. (1997), "Cut problems and their application to divide-and-conquer", in *Approximation algorithms for NP-hard problems*, PWS Publishing Company, Boston, 192–235. *(ND21)*

Shmoys, D. B., Stein, C., and Wein, J. (1994), "Improved approximation algorithms for shop scheduling problems", *SIAM J. Comp.* **23**, 617–632. *(SS17)*

Shmoys, D. B., and Tardos, É. (1993), "Scheduling unrelated machines with costs", *Proc. 4th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 448–454. *(SS6)*

Simchi-Levi, D. (1994), "New worst-case results for the bin-packing problem", *Naval Res. Logistics* **41**, 579–585. *(SR1)*

Simon, H. U. (1989), "Approximation algorithms for channel assignment in cellular radio networks", *Proc. Fundamentals of Computation Theory*, Lecture Notes in Comput. Sci. 380, Springer-Verlag, 405–416. *(MS5)*

Simon, H. U. (1990), "On approximate solutions for combinatorial optimization problems", *SIAM J. Disc. Math.* **3**, 294–310. *(GT15, GT16, AL1)*

Skutella, M. (1997), "Approximation algorithms for the discrete time-cost tradeoff problem", *Proc. 8th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 501–508. *(SS5)*

Slusarek, M. (1989), "A coloring algorithm for interval graphs", *Proc. 14th International Symp. on Mathematical Foundations of Comput. Sci.*, Lecture Notes in Comput. Sci. 379, Springer-Verlag, 471–480. *(SR3)*

Srinivasan, A. (1995), "Improved approximations of packing and covering problems", *Proc. 27th Ann. ACM Symp. on Theory of Comp.*, ACM, 268–276. *(SP4, MP3, MP4)*

Srivastav, A., and Stangier, P. (1994), "Tight approximations for resource constrained sheduling problems", *Proc. 2nd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 855, Springer-Verlag, 307–318. *(SS8)*

Tamir, A. (1991), "Obnoxious facility location on graphs", *SIAM J. Disc. Math.* **4**, 550–567. *(ND54)*

Tarhio, J., and Ukkonen, E. (1988), "A greedy approximation algorithm for constructing shortest common superstrings", *Theoretical Computer Science* **57**, 131–145. *(SR5)*

Trevisan, L. (1996), "Positive linear programming, parallel approximation and PCPs", *Proc. 4th Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 1136, Springer-Verlag, 62–75. *(LO12)*

Trevisan, L. (1997), "When Hamming meets Euclid: the approximability of geometric tsp and mst", *Proc. 29th Ann. ACM Symp. on Theory of Comp.*, ACM, . *(ND31)*

Trevisan, L., Sorkin, G. B:, Sudan, M., and Williamson, D. P. (1996), "Gadgets, approximation, and linear programming", *Proc. 37th Ann. IEEE Symp. on Foundations of Comput. Sci.*, IEEE Computer Society, 617–626. *(LO2)*

Turek, J., Schwiegelshohn, U., Wolf, J. L., and Yu, P. S. (1994), "Scheduling paralle tasks to minimize average response time", *Proc. 5th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 112–121. *(SS8)*

Turner, J. S. (1989), "Approximation algorithms for the shortest common superstring problem", *Inform. and Comput.* **83**, 1–20. *(SR5)*

Verbitsky, O. (1994), "On the largest common subgraph problem", Unpublished manuscript. *(GT42)*

Verbitsky, O. (1995), "On the hardness of approximating some optimization problems that are supposedly easier than Max Clique", *Combinatorics, Probability and Computing* **4**, 167–180. *(GT11, GT21, LO12)*

Vishwanathan, S. (1992), "An approximation algorithm for the asymmetric travelling salesman problem with distances one and two", *Inform. Process. Lett.* **44**, 297–302. *(ND30)*

Vishwanathan, S. (1996), "An $o(\log^* n)$ approximation algorithm for the asymmetric p-center problem", *Proc. 7th Ann. ACM-SIAM Symp. on Discrete Algorithms*, ACM-SIAM, 1–5. *(ND48)*

Vizing, V. G. (1964), "On an estimate of the chromatic class of a p-graph", *Diskret. Analiz.* **3**, 23–30. *(GT7)*

Wagner, F., and Wolff, A. (1995), "An efficient and effective approximation algorithm for the map labeling problem", *Proc. 3rd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 979, Springer-Verlag, 420–433. *(MS12)*

Wang, Q., and Cheng, K.H. (1990), "A heuristic algorithm for the k-center problem with cost and usage weights", Technical Report TR #UH-CS-90-15, Computer Science Department, Houston University. *(ND51)*

Wee, T. S., and Magazine, M. J. (1982), "Assembly line balancing as generalized bin packing", *Oper. Res. Lett.* **1**, 56–58. *(SR1)*

Williamson, D. P., Goemans, M. X., Mihail, M., and Vazirani, V. V. (1995), "A primal-dual approximation algorithm for generalized Steiner network problems", , , 435–454. *(ND9)*

Williamson, D. P., Hall, L. A., Hoogeveen, J. A., Hurkens, C. A. J., Lenstra, J. K., and Shmoys, D. B. (1994), "Short shop schedules", Unpublished manuscript. *(SS14, SS15, SS17)*

Wöginger, G. J., and Yu, Z. (1992), "A heuristic for preemptive scheduling with set-up times", *Computing* **49**, 151–158. *(SS9)*

Yannakakis, M. (1979), "The effect of a connectivity requirement on the complexity of maximum subgraph problems", *J. ACM* **26**, 618–630. *(GT27)*

Yu, B., and Cheriyan, J. (1995), "Approximation algorithms for feasible cut and multicut problems", *Proc. 3rd Ann. European Symp. on Algorithms*, Lecture Notes in Comput. Sci. 979, Springer-Verlag, 394–408. *(ND19)*

Yue, M. (1991), "A simple proof of the inequality $MFFD(L) \leq \frac{71}{60} OPT(L) + \frac{78}{71}, \forall L$ for the $MFFD$ bin-pack algorithm", Technical Report RRR # 20-91, Rutcor, Rutgers Center for Operations Research, Rutgers University, New Jersey. *(SR1)*

Zelikovsky, A. Z. (1994), "Better approximation bounds for the network and Euclidean Steiner tree problems", Unpublished manuscript. *(ND8)*

Zhang, K., and Jiang, T. (1994), "Some MAX SNP-hard results concerning unordered labeled trees", *Inform. Process. Lett.* **49**, 249–254. *(GT44)*

Zuckerman, D. (1993), "NP-complete problems have a version that's hard to approximate", *Proc. Eight Ann. Structure in Complexity Theory Conf.*, IEEE Computer Society, 305–312. *(GT1, GT5, GT8, GT9, GT15, GT38, ND7, ND14, SP1, SP4, SP7, SP9, SS1)*

Index