PHY 2049 Exam 3 Solutions

Problem 1:

A charged particle, **Q = 0.5 C**, enters a region with uniform electric and magnetic fields given by $\vec{E} = E_x\hat{x} + E_z\hat{z}$ and $\vec{B} = B_x\hat{x} + B_y\hat{y}$, where **$E_x$ = 2 V/m, E_z = -10 V/m, B_x = 1 T, B_y = 2 T**. What is the magnitude of the *net* force on the particle (**in N**) at the moment its velocity is given by $\vec{v} = v_x\hat{x}$ with **v_x = 5 m/s**?

Answer: 1

Solution: The magnetic force on the particle is given by

$$\vec{F}_B = Q\vec{v}\times\vec{B} = Q\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & 0 & 0 \\ B_x & B_y & 0 \end{vmatrix} = Qv_xB_y\hat{z}$$

and the net force is

$$\begin{aligned} \vec{F} &= \vec{F}_E + \vec{F}_B = Q(\vec{E} + \vec{v}\times\vec{B}) = Q(E_x\hat{x} + (E_z + v_xB_y)\hat{z}) \\ &= (0.5C)(2V/m)\hat{x} = (1V/m)\hat{x} \end{aligned}.$$

Problem 2:

Two particles are undergoing circular motion in a uniform magnetic field that is perpendicular to their velocities. Particle 1 has a charge of **0.1 C**, a mass of **2 grams**, and a radius for its circular motion of **0.5 m**. Particle 2 has a charge of **0.2 C**, a mass of **4 grams**, and a radius for its circular motion of **2 m**. If it takes particle 1 one second to make one complete revolution, how long (**in s**) does it take particle 2 to make one complete revolution?

Answer: 1

Solution: For circular motion in a uniform magnetic field we have,

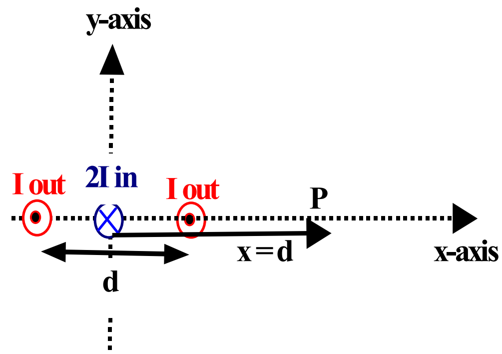
$$F = qvB = mv^2/R, \quad v = \frac{qBR}{m}, \text{ and } T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}.$$

Thus,

$$T_1 = \frac{2\pi m_1}{q_1B} \text{ and } T_2 = \frac{2\pi m_2}{q_2B} \text{ and } T_2 = \frac{q_1m_2}{q_2m_1}T_1 = \frac{(0.1C)(4g)}{(0.2C)(2g)}T_1 = T_1.$$

The two particles have the same period and thus **T_2= T_1 = 1s**.

Problem 3:



Three infinitely long parallel wires lie in the xz-plane and are located at **x = -d/2**, **x = 0**, and **x = d/2** as shown in the **Figure**. The two outside wires (x = d/2,-d/2) carry equal currents **I** (*out of page*), while the center wire (x = 0) caries a current of **2I** (*into the page*). What is the magnitude of the magnetic field at the point **x = d** on the x-axis ($k = \mu_0/4\pi$)?

Answer: 4kI/(3d)

Solution: The magnetic field at the point **x > d/2** on the x-axis is the superposition of the fields from each of the three wires as follows:

$$\vec{B}(x) = \left(\frac{2kI}{x+d/2} + \frac{2kI}{x-d/2} - \frac{2k(2I)}{x}\right)\hat{y} = \frac{kId^2}{x(x^2-(d/2)^2)}\hat{y},$$

and setting **x = d** gives

$$\vec{B}(d) = \frac{kId^2}{d(d^2-(d/2)^2)}\hat{y} = \frac{4kI}{3d}\hat{y}.$$

Problem 4:

In the previous problem, what is the x component of the *net* force exerted on a length **L** of the wire at **x = d/2** due to the other two wires ($k = \mu_0/4\pi$)?

Answer: $6kI^2L/d$

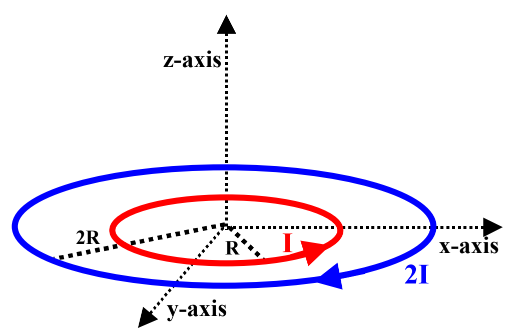
Solution: The magnetic field at **x = d/2** due to the other two wires is as follows:

$$\vec{B}_{12} = \left(\frac{2kI}{d} - \frac{2k(2I)}{d/2}\right)\hat{y} = -6\frac{kI}{d}\hat{y},$$

and the force on wire 3 is thus,

$$\vec{F} = I\vec{L}\times\vec{B}_{12} = -6\frac{kI^2L}{d}\hat{z}\times\hat{y} = 6\frac{kI^2L}{d}\hat{x}.$$

Problem 5:



Two concentric current loops lie in the xy plane and have radii **R**, **2R**, as shown in the **Figure**. The loop with radius **R** carries a current **I** in the counter-clockwise direction and the loop with radius **2R** carries a current of **2I** in the opposite direction (*i.e.* clockwise). What is the z component of the *net* magnetic field at the center of the two loops (*i.e.* at the point z = 0 on the z-axis, $k = \mu_0/4\pi$)?

Answer: zero

Solution: At the center of the circles the magnetic field is the superposition of the fields from each of the two loops as follows:

$$\vec{B} = \left(\frac{2\pi kI}{R} - \frac{2\pi k(2I)}{2R}\right)\hat{z} = 0\,.$$

Problem 6:

In the previous problem, what is the z component of the *net* magnetic field at the point **z = R** on the z-axis (k = $\mu_0/4\pi$)?

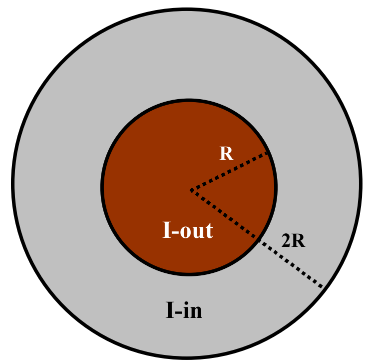
Answer: -2.3kI/R

Solution: At the a point **z** on the z-axis the magnetic field is the superposition of the fields from each of the two loops as follows:

$$\vec{B}(z) = \left(\frac{2kI\pi R^2}{(z^2+R^2)^{3/2}} - \frac{2k(2I)\pi(2R)^2}{(z^2+(2R)^2)^{3/2}}\right)\hat{z} = 2\pi kI\left(\frac{R^2}{(z^2+R^2)^{3/2}} - \frac{8R^2}{(z^2+4R^2)^{3/2}}\right)\hat{z}$$

$$\xrightarrow[z=R]{} \frac{2\pi kI}{R}\left(\frac{1}{(2)^{3/2}} - \frac{8}{(5)^{3/2}}\right)\hat{z} = \frac{2\pi kI}{R}\left(\frac{\sqrt{2}}{4} - \frac{8\sqrt{5}}{25}\right)\hat{z} = -2.27\frac{kI}{R}\hat{z}$$

Problem 7:



An infinitely long coaxial cable consists of an inner wire of radius **R** carrying a uniformly distributed current **I** (*out of the page*) surrounded by an outer wire with inner radius **R** and outer radius **2R** carrying a uniformly distributed current **I** (*into the page*) as shown in the **Figure**. For both wires the current density is constant? What is the magnitude of the *net* magnetic field at a point **r > 2R** outside both wires (k = $\mu_0/4\pi$)?

Answer: zero

Solution: Ampere's Law tell us that

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$

and if we take a circular loop with radius **r > 2R** then $\mathbf{I_{enclosed} = 0}$ and hence **B = 0**.

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Problem 8:

In the previous problem, what is the magnitude of the *net* magnetic field at the point **r = 3R/2** inside the outer wire (k = $\mu_0/4\pi$)?

Answer: 7kI/(9R)

Solution: If we take a counter-clockwise circular loop with radius **r** such that **R < r < 2R** then

$$\oint \vec{B}\cdot d\vec{l} = 2\pi r B(r) = \mu_0 I_{enclosed} = \mu_0 I - \mu_0 J(\pi r^2 - \pi R^2) = \mu_0 \frac{4R^2 - r^2}{3R^2} I\,,$$

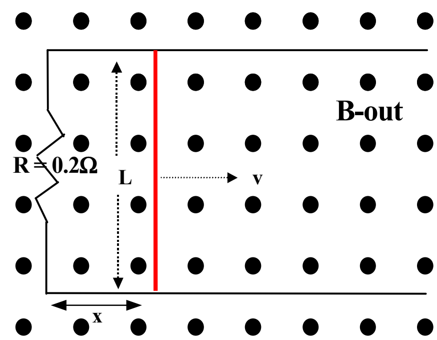
where I used

$$I = J(\pi(2R)^2 - \pi R^2) = J\pi 3R^2\,,$$

and where **J** is the current density (*constant*) of the outer wire. Thus,

$$B(r) = \frac{2kI}{r}\left(\frac{4R^2 - r^2}{3R^2}\right) \xrightarrow[r=3R/2]{} \frac{2kI}{3R/2}\left(\frac{4R^2 - (3R/2)^2}{3R^2}\right) = \frac{7kI}{9R}.$$

Problem 9:



A moveable (massless and frictionless) bar with a length of **L = 0.5 meters** is being moved along two conducting rails at a constant velocity, **v = 2 m/s**, as shown in the **Figure**. The entire system is immersed in a uniform magnetic field (*out of the paper*) with magnitude $\mathbf{B(t)=B_0e^{-t/\tau}}$, where $\mathbf{B_0 = 2}$ **T** and $\boldsymbol{\tau}$ **= 1 second**. If the bar is at **x = 0** at **t = 0**, what is the magnitude of the induced EMF in the circuit (**in Volts**) at the time **t = 1 second**?

Answer: zero

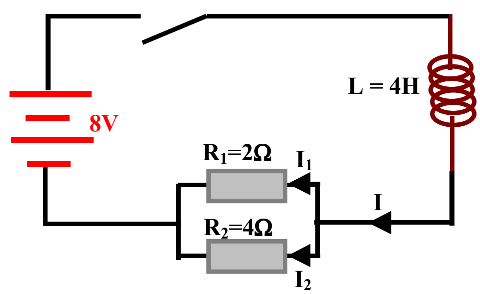
Solution: If I choose my orientation counter-clockwise then the magnetic flux through the loop is given by

$$\Phi_B = B(t)xL = B_0e^{-t/\tau}vtL\ ,$$

where I used **x = vt**. From Faraday's Law of induction the induced EMF is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -vLB_0e^{-t/\tau}(1 - t/\tau)\ ,$$

which vanishes at t = 1 second since $\mathbf{1-t/\tau = 0}$ at t = 1s.



Problem 10:

Consider the LR circuit shown in the **Figure** which consists an **8 Volt EMF**, an inductor, **L = 4 H**, and two resistors. After the switch is closed, what is the stored energy in the inductor (**in Joules**) at the instant the current through the **2 Ω** resistor is equal to **2 Amps**?

Answer: 18

Solution: The current through the inductor is given by $\mathbf{I = I_1+I_2}$ as shown in the figure, where the current through the **2 Ω** resistor is

$$I_1 = \frac{R_2}{R_1 + R_2}I \quad \text{so that} \quad I = \frac{R_1 + R_2}{R_2}I_1 = \frac{6}{4}(2A) = 3A\ .$$

The stored energy in the inductor is given by

$$U = \tfrac{1}{2}LI^2 = \tfrac{1}{2}(4H)(3A)^2 = 18J\ .$$